Why Chance Constrains Credence

David Builes and Jack Spencer

Draft as of: 06/19/18

Words: 7,714

Words, including notes and references: 8,577

§1/ Introduction

Chance-talk is ubiquitous in science, both in fundamental science and in the special sciences, and no less common in nonscientific contexts. We talk about the chance of winning the lottery, the chance of a defendant being found guilty, the chance of a politician winning reelection, and so on.

The claims we make about chance are as diverse as the matters about which we make them. Some claims are about exact values:

1. The chance of rolling double-six is 1/36.
2. A tritium atom has a 0.5 chance of decaying within 12.26 years.

Some claims are about approximate or comparative values:

3. There's about a 30% chance that tomorrow's baseball game will get rained out.

4. The New York Yankees have a better chance of winning the 2018 World Series than do the Boston Red Sox.[1]

Some claims are only about what alters the chances:

5. Eating broccoli decreases the chance of developing bladder cancer.[2]
6. Your chance of going on a second date with someone increases if you take them to sushi on the first date.[3]

There are metaphysical differences—sometimes stark, very important metaphysical differences—among the things we call 'chance'. Some of what we call 'chance' is trivialized by determinism, and some is not. Some of what we call 'chance' is bound up with the fundamental indeterminacies of quantum mechanics; some is bound up instead with the laws of the special sciences; and some is not bound up with any laws of nature—there are no laws concerning dating or baseball, for example. Some of what we call 'chance' is bound up with frequencies or frequency-explaining propensities, but other things we call 'chance', like the chance of the Democrats having a majority of the seats in the U.S. Senate in 2045, are mere one-off affairs. Some of what we call 'chance', like the chance of rolling double-six, is bound up with

[1] https://projects.fivethirtyeight.com/2018-mlb-predictions/?ex_cid=rrpromo

[2] https://www.webmd.com/cancer/bladder-cancer/news/20080228/broccoli-sprouts-vs-bladder-cancer

[3] https://www.esquire.com/lifestyle/sex/news/a41698/match-singles-in-america-study/

symmetries, but some is not. It is not by virtue of any symmetry that the Houston Astros have a 22% chance of winning the 2018 World Series.[4]

Many philosophers take the metaphysical disunity of the things we call ‘chance’ to be a good reason to narrow the subject matter.[5] In their view, what we talk about when we talk about chance is sometimes chance*,* the proper topic of philosophical theory, and sometimes something else, some sort of counterfeit chance.[6]

We are happy to let a thousand flowers bloom. Since there are metaphysical differences among the the things we call ‘chance’, there is no harm in using one to mark some divide between chance and counterfeit chance. When it comes to the metaphysics of chance, drawing such a distinction might prove useful.

But when it comes to understanding the epistemic import of chance, which is our main concern in this essay, drawing a distinction along metaphysical lines between chance and counterfeit chance is liable to be counterproductive; for the things we call ‘chance’, though metaphysically disunified, are epistemically unified. Chance constrains credence. An agent’s credences cannot be rational unless they conform in the right way to what the agent knows about the chances. And the things we call ‘chance’ all constrain credence in the same, special way.

Our goal in this essay is to explain why chance constrains credence, while allowing chance to be as metaphysically disunified as it appears to be. Those who distinguish chance and

---

[4] https://projects.fivethirtyeight.com/2018-mlb-predictions/?ex_cid=rrpromo, as of June 13, 2018.

[5] See *e.g.* Emery (2017), Lewis (1980), Popper (1982), Schaffer (2007).

[6] *Cf.* Lewis (1986: 120). Also see *e.g.* Loewer (2001) and Schaffer (2007).

counterfeit chance can understand us as offering an explanation for why chance-and-counterfeit-chance constrains credence.

The ways that chance constrains credence are set forth by the various *chance-credence norms*. Explaining why chance constrains credence is explaining why chance-credence norms are true.

Although there has been quite a lot of philosophical work devoted to the matter, there is not, at this point, any consensus about why chance-credence norms are true.[7] And some philosophers despair: They think that it is impossible to explain chance-credence norms, that chance-credence norms either must be rejected or accepted as primitive, additional constraints on rationality.[8]

Our optimism that chance-credence norms can be explained stems from recent progress made elsewhere in epistemology. The special way that chance constrains credence is unique—save for one notable exception. Credence constrains itself in the same, special way. Just as there are chance-credence norms, which specify the various ways in which an agent is rationally required to conform her credences to what she knows about the chances, there are *reflection principles*, which specify the various ways in which an agent is rationally required to conform her credences to what she knows about her own (future) credences.[9] Reflection

[7] See *e.g.* Bigelow and Pargetter (1990), Elliott (2016), Gallow (MS), Howson and Urbach (1989), Hoefer (2007), Lange (2017), Loewer (2001), Mellor (1971), Pettigrew (2012; 2013; 2016), Rayo (MS), and Schwarz (2014).

[8] See *e.g.* Black (1998), Hall (2004), and Strevens (1999).

[9] *Cf.* van Fraassen (1984).

principles are not mystifying. They were thought to be, but Briggs (2009a) and Weisberg (2007) corrected that misimpression. As Briggs and Weisberg (2007) showed, reflection principles, when formulated properly, are theorems of the probability calculus. Thus, far from being primitive, additional constrains on rationality, reflection principles are really just reminders that rationality requires credences to be probabilistic.

We want to do for chance-credence norms what Briggs and Weisberg have done for reflection principles. To that end, we defend the *epistemic theory of chancemaking*. As we will show, if the epistemic theory of chancemaking is true, then every true chance-credence norm follows from the probability axioms. Far from being primitive, additional constrains on rationality, chance-credence norms are really just reminders that rationality requires credences to be probabilistic.

Here is a brief roadmap for the essay. In sections 2-4, we develop the epistemic theory of chancemaking as applied to non-Humean chance. In section 5, we extend the discussion to Humean chance. In section 6, we argue that the epistemic theory of chancemaking has important consequences for the question of which priors are rational. In section 7 and 8, we consider some objections and conclude.

§2/ The Epistemic Theory of Chancemaking

Many philosophers agree that truth-value facts are never fundamental; as the saying goes, truth depends on being.[10] The truth-value facts are facts about the alethic properties had by propositions. Alethic properties are properties had contingently by propositions, and, like many

[10] See *e.g.* Armstrong (2005) and Bigelow (1988).

philosophers, we think that the contingent properties had by propositions are always had derivatively, by virtue of some facts about worldly affairs. The proposition that snow is white is true, for example, because snow—the worldly stuff, itself—is white.

As one of our main assumptions, we claim that chance facts are never fundamental. The chance facts are facts about probabilistic properties and relations had by propositions. These probabilistic properties and relations are had by propositions contingently, and thus, we claim, derivatively, by virtue of some facts about worldly affairs. This is not to say that the chance facts supervene on the Humean mosaic. One way that chance facts could be derivative is by holding in virtue of facts about the Humean mosaic, but that is not the only way. In fact, we suspect that chance facts do not supervene on the Humean mosaic. We are inclined to think at least some chance facts hold in virtue of the world containing non-Humean whatnots: laws, powers, propensities, or some such. At this point, it is not important that we decide which facts about worldly affairs give rise to the chance. All that we need to know is that chance facts are never fundamental.

There are various kinds of chance: the chances in statistical mechanics, the chances in biology, the chances in quantum theory, and so on. Let the *K*-chances be some particular kind. There then will be various *K-chance propositions.* Among them:

- $\langle \mathrm{Ch}_K(P) = x \rangle$: the proposition that the *K*-chance of *P* equals *x*.
- $\langle \mathrm{Ch}_K(P) \approx x \rangle$: the proposition that the *K*-chance of *P* approximates *x*.
- $\langle \mathrm{Ch}_K(P) < \mathrm{Ch}_K(P|Q) \rangle$: the proposition that the *K*-chance of *P* is less than the *K*-chance of *P* conditional on *Q*.

- $\langle \mathrm{Ch}_K(P) = (2)(\mathrm{Ch}_K(Q))\rangle$: the proposition that the *K*-chance of *P* is twice the *K*-chance of *Q*.

The *K-chance facts* at some possible world *w* are the *K*-chance propositions true at *w*.

Since the *K*-chance facts are not fundamental, at any world *w*, there will be some worldly fact, which nowhere mentions chance or probability, which makes the *K*-chances be what they are at *w*. We call that fact the *K-chancemaker* at *w*. The *K*-chancemaker at *w* is the *chancemaker* for every *K*-chance proposition that holds true at *w*.[11]

For each *K*-chance proposition, we can identify the set of its *possible chancemakers*, a set that contains its chancemaker at every world at which it is true. For example, suppose that at any world w_i, the *K*-chancemaker at w_i is K_{wi}, and suppose that $\langle \mathrm{Ch}_K(P) = x\rangle$ is true at worlds w_1, w_2, … Then, $\{K_{w1}, K_{w2}, \ldots\}$ is the set of possible chancemakers of $\langle \mathrm{Ch}_K(P) = x\rangle$.

A theory of *chance* purports to explain why the chances are what they are. The chances are what they are in virtue of the possible chancemakers that obtain, so it is incumbent on a theory of chance to identify what the possible chancemakers are.

The epistemic theory of chancemaking is not a theory of chance. It does not purport to identify what the possible chancemakers are. Rather, it is a theory of *chancemaking*. It purports to explain why possible chancemakers are possible chancemakers.

Suppose that K_w is a possible chancemaker for $\langle \mathrm{Ch}_K(P) = x\rangle$. Then, according to the epistemic theory,

---

[11] If the *K*-chances are time-relative, then for each time *t*, there will be some K_t-chancemaker that makes the *K*-chances be what they are at time *t*.

> K_w is a possible chancemaker for $\langle \mathrm{Ch}_K(P) = x \rangle$ because every rational prior, when updated on K_w, reflects $\langle \mathrm{Ch}_K(P) = x \rangle$.

And, in general, if C is some chance proposition and M is some nonchance proposition, then, according to the epistemic theory of chancemaking:

> If M is a possible chancemaker for C, then it is so because every rational prior, when updated on M, reflects C.[12]

To better understand (12), the core of the epistemic theory of chancemaking, we need to know what a rational prior is and what it is for a rational prior, when updated on M, to reflect C.

A *rational prior* is any probability function that a rational agent could have as her credences in the absence of any evidence. There is considerable disagreement about which probability functions are rational priors. Subjective Bayesians claim that every probability function is a rational prior. Moderate Bayesians claim that many probability functions are, and many probability functions are not, rational priors. Objective Bayesians claim that only one probability function is a rational prior. We will return to the dispute about which probability functions are rational priors in section 6. For now, we will ignore it and assume nothing about rational priors except that they are probability functions.

---

[12] One could also defend the reverse entailment: that M is a possible chancemaker for C if every rational prior, when updated on M, reflects C. In this essay, we take no stand on the reverse entailment.

A rational prior updated on *M* is a rational prior conditionalized on *M*. For example, if $C_0(-)$ is a rational prior, then $C_0(-|M)$ is a rational prior updated on *M*. And $C_0(-|M)$ *reflects* a chance proposition just if, according to $C_0(-|M)$, the probabilistic properties and relations among propositions are as *C* alleges them to be. For example, to return to *K*-chance, $C_0(-|K_w)$ reflects $\langle Ch_K(P) = x \rangle$ just if $C_0(P|K_w) = x$, and $C_0(-|K_w)$ reflects $\langle Ch_K(P) < Ch_K(P|Q) \rangle$ just if $C_0(P|K_w) < C_0(P|Q\&K_w)$.

The epistemic theory of chancemaking explains both the metaphysical disunity and the epistemic unity of chance. Chance is metaphysically disunified because the possible chancemakers are disunified. The possible chancemakers are not queer, in Mackie's sense.[13] They are the facts that we expect them to be: the ordinary, descriptive facts that people who know about the chances investigate. But different kinds of chance have different kinds of possible chancemakers. For some kinds of chance, the possible chancemakers are laws or laws together with initial segments of history. For other kinds of chance, the possible chancemakers are propensities or frequencies or symmetries, or some combination thereof. For still other kinds of chance, like the chance of the Houston Astros winning the 2018 World Series, the possible chancemakers are unsurveyably complicated, intertwining matters of fact. We suspect that there is no simple account of what the possible chancemakers are.

But there *is* a simple account of why possible chancemakers are possible chancemakers: namely, the epistemic theory. The epistemic import of chance is explained, not by an account of what the possible chancemakers are, but by an account of why possible chancemakers are

[13] *Cf.* Mackie (1977).

possible chancemakers. The epistemic theory of chancemaking thus explains why chance is epistemically unified, while allowing chance to be metaphysically disunified.

§3/ Fundamental Chance

Most of what we call 'chance' are various sorts of nonfundamental chance, but it will be helpful to start with fundamental chance.

Fundamental chance is time-relative. For any time t, there are various *t-chance propositions*. Among them:

- $\langle \text{Ch}_{F,t}(P) = x \rangle$: the proposition that the t-chance of P equals x.
- $\langle \text{Ch}_{F,t}(P) \approx x \rangle$: the proposition that the t-chance of P approximates x.
- $\langle \text{Ch}_{F,t}(P) < \text{Ch}_{F,t}(P|Q) \rangle$: the proposition that the t-chance of P is less than the t-chance of P conditional on Q.
- $\langle \text{Ch}_{F,t}(P) = (2)(\text{Ch}_{F,t}(Q)) \rangle$: the proposition that the t-chance of P is twice the t-chance of Q.

The *t-chance facts* at some world w are the t-chance propositions that are true at w.

Following the orthodoxy, we take the t-chances to derive from the fundamental laws of nature and the history of the world up to time t. Let L_w be the *fundamental law proposition* that fully specifies the fundamental laws at world w. Let H_{tw} be the *t-history proposition* that fully specifies the history of world w up to time t. Let $\boldsymbol{LH_t}$ be the set containing every possible conjunction of a fundamental law proposition and a t-history proposition, a set that partitions the space of possible worlds. The members of $\boldsymbol{LH_t}$ are the *possible t-chancemakers*. At any world w, the member of $\boldsymbol{LH_t}$ that is true at w is the chancemaker for every t-chance proposition that holds at w. As shorthand, we will write "LH_{tw}" for "L_wH_{tw}" below.

Each t-chance proposition is necessarily equivalent to the disjunction of its possible chancemakers, and we think that the following argument shows that this necessary equivalence is knowable *a priori*. Suppose that LH_{tw1}, LH_{tw2}, ... are the possible chancemakers of $\langle \mathrm{Ch}_{F,t}(P) = x \rangle$. Since the epistemic theory of chancemaking is knowable *a priori*, it is knowable *a priori* that $\langle \mathrm{Ch}_{F,t}(P) = x \rangle$ is true at a possible world w if and only if the member of $\boldsymbol{LH_t}$ that holds at w is such that every rational prior, when updated on it, reflects $\langle \mathrm{Ch}_{F,t}(P) = x \rangle$. The facts about the nature of rationality are knowable *a priori*.[14] Therefore, it is knowable *a priori* that every rational prior, when updated on LH_{tw1}, gives x probability to P; it is knowable *a priori* that every rational prior, when updated on LH_{tw2}, gives x probability to P; …; and it is knowable *a priori* that there is no other member of $\boldsymbol{LH_t}$ which is such that every rational prior, when updated on it, gives x probability to P. We thus arrive at the our desired conclusion: It is knowable *a priori* that $\langle \mathrm{Ch}_{F,t}(P) = x \rangle$ and $LH_{tw1} \vee LH_{tw2} \vee$ ... are necessarily equivalent. By parallel reasoning, we can show that the necessary equivalence between *any* t-chance proposition and the disjunction of its possible chancemakers is knowable *a priori.*

There are *lots* of chance-credence norms that hold for fundamental chance, many more than philosophers typically realize. We have divided them into three families. The first family, we call:

[14] Here we follow *e.g.* Littlejohn (2018) and Titelbaum (2015).

Initial Principles:

(a) For any rational prior $C_0(\text{-})$, $C_0(P|\langle Ch_{F,t}(P) = x\rangle) = x$, if defined.

(b) For any rational prior $C_0(\text{-})$, $C_0(P|\langle Ch_{F,t}(P) \approx x\rangle) \approx x$, if defined.

(c) For any rational prior $C_0(\text{-})$, $C_0(P|\langle Ch_{F,t}(P) < Ch_{F,t}(P|Q)\rangle) <$ $C_0(P|Q\&\langle Ch_{F,t}(P) < Ch_{F,t}(P|Q)\rangle)$, if both are defined.

(d) For any rational prior $C_0(\text{-})$, $C_0(P|\langle Ch_{F,t}(P) = (2)(Ch_{F,t}(Q))\rangle) =$ $(2)(C_0(Q|\langle Ch_{F,t}(P) = (2)(Ch_{F,t}(Q))\rangle)$, if both are defined.

(e) …

In effect, the Initial Principles say that an agent, in the absence of any other information, is rationally required to conform her credences to what she knows about the *t*-chances.

The second family, we call:

Evidential Principles:

(a) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P|E) = x\rangle) = x$, if defined.

(b) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P|E) \approx x\rangle) \approx x$, if defined.

(c) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P|E) < Ch_{F,t}(P|E\&Q)\rangle) <$ $C_0(P|E\&Q\&\langle Ch_{F,t}(P|E) < Ch_{F,t}(P|E\&Q)\rangle)$, if both are defined.

(d) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P|E) = (2)(Ch_{F,t}(Q|E))\rangle) =$ $(2)(C_0(Q|E\&\langle Ch_{F,t}(P|E) = (2)(Ch_{F,t}(Q|E))\rangle))$, if both are defined.

(e) …

In effect, the Evidential Principles say that an agent is rationally required to conform her credences to what she knows about the *t*-chances conditional on her total evidence.

The third family, which includes Lewis's Principal Principle,[15] we call:

Resiliency Principles:[16]

(a) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P) = x\rangle) = x$, if E is admissible with respect to $\langle Ch_{F,t}(P) = x\rangle$.

(b) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P) \approx x\rangle) \approx x$, if E is admissible with respect to $\langle Ch_{F,t}(P) \approx x\rangle$.

(c) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P) < Ch_{F,t}(P|Q)\rangle) < C_0(P|E\&Q\&\langle Ch_{F,t}(P|E) < Ch_{F,t}(P|Q)\rangle)$, if E is admissible with respect to $\langle Ch_{F,t}(P) < Ch_{F,t}(P|Q)\rangle$.

(d) For any rational prior $C_0(\text{-})$, $C_0(P|E\&\langle Ch_{F,t}(P) = (2)(Ch_{F,t}(Q))\rangle) = (2)(C_0(Q|E\&\langle Ch_{F,t}(P) = (2)(Ch_{F,t}(Q))\rangle)$, if E is admissible with respect to $\langle Ch_{F,t}(P) = (2)(Ch_{F,t}(Q))\rangle$.

(e) …

[15] *Cf*. Lewis (1980).

[16] *Cf*. Skyrms (1977).

In effect, the Resiliency Principles say that, so long as an agent's total evidence is *admissible*—a notion we will characterize momentarily—the agent is rationally required to conform her credences to what she know about the *t*-chances.

Working in a coarse-grained,[17] countable framework, we can show that each of these chance-credence norms is a theorem of the probability calculus. We need only the two premises already introduced: the epistemic theory of chancemaking and the claim that each *t*-chance proposition is *a priori* equivalent to the disjunction of its possible chancemakers.[18]

Take Initial Principle (a). Suppose that $C_0(P|\langle Ch_{F,t}(P) = x\rangle)$ is defined, and suppose that the possible chancemakers of $\langle Ch_{F,t}(P) = x\rangle$ are $LH_{tw1}, LH_{tw2}, \ldots$. Every *t*-chance is *a priori* equivalent to the disjunction of its possible chancemakers, and every rational prior is certain of all *a priori* truths. So, if $C_0(\text{-})$ is a rational prior,

$$C_0(P|\langle Ch_{F,t}(P) = x\rangle) = C_0(P|LH_{tw1} \vee LH_{tw2} \vee \ldots).$$

[17] The setting is coarse-grained setting because chance is coarse-grained. The chance that Clark Kent is *F* is necessarily equal to the chance that Superman is. Familiar puzzles about guises arise whenever one works in a coarse-grained setting. But those puzzles are independent of, and orthogonal to, our discussion here, so we will be ignoring them.

[18] Our strategy for deriving chance-credence norms as applied to fundamental chance from the probability axioms is a variation on the one in Rayo (MS). Our variation allows us to extend Rayo's strategy to nonfundamental chances and to other objective probabilities (like Humean superchances) that constrain credence.

Because the possible chancemakers are mutually exclusive,

$$C_0(P|LH_{tw1} \vee LH_{tw2} \ldots) = C_0(P|LH_{tw1})C_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots .$$

Now the crucial step: Because the epistemic theory of chancemaking is true of fundamental chance, any rational prior, when updated on any possible chancemaker of $\langle Ch_{F,t}(P) = x\rangle$, assigns x probability to P. Therefore, each $C_0(P|LH_{twi})$ equals x. Therefore,

$$C_0(P|LH_{tw1})C_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots = xC_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots .$$

Distributing multiplication over addition,

$$xC_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots = x(C_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots).$$

The $C_0(LH_{twi}|LH_{tw1} \vee LH_{tw2} \vee \ldots)$'s sum to one. So, by the law of total conditional probability,

$$x(C_0(LH_{tw1}|LH_{tw1} \vee LH_{tw2} \vee \ldots) + \ldots) = x(1) = x.$$

And that concludes the proof. Similar proofs show that the other Initial Principles are also theorems of the probability calculus.

Turn, now, to Evidential Principle (a). To begin the proof, suppose that $C_0(P|E\&\langle Ch_{F,t}(P|E) = x\rangle)$ is defined, and suppose that the possible chancemakers for $\langle Ch_{F,t}(P) = x\rangle$ are $LH_{tw1}, LH_{tw2}, \ldots$. If C_0(-) is a rational prior, then:

$$C_0(P|E\&\langle Ch_{F,t}(P|E) = x\rangle) = C_0(P|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots)).$$

Because the possible chancemakers are mutually exclusive,

$$C_0(P|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) = C_0(P|E\&LH_{tw1})C_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots)) + \ldots.$$

According to the epistemic theory, a member of $\boldsymbol{LH_t}$ is a possible chancemaker of $\langle Ch_{F,t}(P|E) = x\rangle$ if and only if every rational prior, when updated on the conjunction of it and E, assigns x probability to P. Therefore, each $C_0(P|E\&LH_{tw1})$ equals x. Therefore,

$$C_0(P|E\&LH_{tw1})C_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots)) + \ldots = xC_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) + \ldots .$$

Distributing multiplication over addition and applying the law of total conditional probability,

$$xC_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) + \ldots = x(C_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) + \ldots) = x(C_0(E\&LH_{tw1}|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) + \ldots) = x(1) = x.$$

And that concludes the proof. Similar proofs show that the other Evidential Principles are also theorems of the probability calculus.

We turn finally to the Resiliency Principles. To prove the Resiliency Principles, we need to characterize admissibility. Like Meacham (2010), we think that non-Humeans should accept the following definition of admissibility: A proposition E is *admissible* with respect to some t-chance proposition C if and only if every possible chancemaker of C that is compatible with E entails E. We can think of this characterization intuitively. An agent should defer to a probability function only if the probability function is "more informed" than the agent is, and a probability function that knows some t-chance proposition is more informed than the agent just if every one of its possible chancemakers that is compatible with the agent's evidence entails the agent's evidence.

Take Resiliency Principle (a), better known as the Principal Principle. Suppose that $C_0(P|E\&\langle Ch_{F,t}(P) = x\rangle)$ is defined, and suppose that the possible chancemakers of $\langle Ch_{F,t}(P) = x\rangle$ are $LH_{tw1}, LH_{tw2}, \ldots$. If $C_0(\text{-})$ is a rational prior, then:

$$C_0(P|E\&\langle Ch_{F,t}(P) = x\rangle) = C_0(P|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots).$$

Let $LH_{twi}, LH_{twj}, \ldots$ be the possible chancemakers of $\langle Ch_{F,t}(P) = x\rangle$ that are compatible with E. Since E is admissible with respect to $\langle Ch_{F,t}(P) = x\rangle$, each of $LH_{twi}, LH_{twj}, \ldots$ entails E. So,

$$C_0(P|E\&LH_{tw1} \vee E\&LH_{tw2} \vee \ldots) = C_0(P|LH_{twi} \vee LH_{twj} \vee \ldots).$$

Because the disjuncts are mutually exclusive,

$$C_0(P|LH_{twi} \vee LH_{twj} \vee ...) = C_0(P|LH_{twi})C_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots \, .$$

Because the epistemic theory is true, each $C_0(P|LH_{twi})$ equals x. So,

$$C_0(P|LH_{twi})C_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots = xC_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots \, .$$

Distributing multiplication over addition and applying the law of total conditional probability,

$$xC_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots = x(C_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots =$$
$$x(C_0(LH_{twi}|LH_{twi} \vee LH_{twj} \vee ...) + \ldots) = x(1) = x.$$

And that concludes the proof. The Principal Principle is a theorem of the probability calculus, and similar proofs apply to the other Resiliency Principles.

§4/ Nonfundamental Chance

Fundamental chance plays little or no role in our lives. We often conform our credence to what we know about the chances. But, since the fundamental theory of physics is yet be discovered, it is unclear whether anyone has ever known anything about fundamental chance, and thus it is unclear whether anyone has ever conformed their credence to something they have known about the fundamental chances. For the most part, it is nonfundamental chance that constrains credence. An explanation of why chance constrains credence therefore must extend to nonfundamental chance.

Our proposed extension always follows the same, three-step process.[19] *Step #1:* For any kind of chance, *K*-chance, we find an associated partition, the elements of which are the possible chancemakers for that kind of chance. *Step #2:* We claim that *K*-chance propositions are *a priori* equivalent to the disjunction of their possible chancemakers, where their possible chancemakers are elements of the associated partition. *Step #3:* We claim that the epistemic theory of chancemaking holds of *K*-chance, that every rational prior, when updated on any possible chancemaker of a *K*-chance proposition, reflects the *K*-chance proposition. Taken together, these three claims ensure that *K*-chance constrains credence in the same, special way that fundamental chance does.

Take statistical mechanical chance, for example. The possible chancemakers for statistical mechanical chance are conjunctions of laws and initial segments of macro history. Let M_{tw} be a *macro t-history proposition*, a full description of the macro history of the world up to time *t*. Let $\boldsymbol{LM_t}$ be the set containing every possible conjunction of a law proposition and a macro *t*-history proposition. *Step #1:* The members of $\boldsymbol{LM_t}$ are the possible statistical mechanical chancemakers. *Step #2*: Every statistical mechanical chance proposition is *a priori* equivalent to the disjunction of the members of $\boldsymbol{LM_t}$ that are its chancemakers. *Step #3*: The epistemic theory of chancemaking holds of statistical mechanical chance.

[19] This conception of nonfundamental chance draws on Handfield and Wilson's (2014) generalized chance, which in turn draws on Meacham (2005; 2010a).

Taken together, these claims ensure that every chance-credence norms that is true of fundamental chance—specifically, the Initial Principles, the Evidential Principle, and the Resiliency Principle—are also true of statistical mechanical chance. They also ensure that every chance-credence norms that holds for statistical mechanical chance is a theorem of the probability calculus. The proofs are like the ones above, except they contain statistical mechanical *t*-chance propositions in place of *t*-chance propositions and LM_{tw}'s in place of LH_{tw}'s.

Applying the same three-step process to every kind of nonfundamental chance we ensure (i) that every chance-credence norms that is true of fundamental chance is also true of every kind of nonfundamental chance, and (ii) that every chance-credence norms that is true of any kind of nonfundamental chance is a theorem of the probability calculus.

In ordinary conversation, speakers rarely specify what kind of chance they are talking about. They simply talk about chance. For example, they might say:

7. There's a good chance that *P*.
8. There's a chance that *Q*.
9. This oddly shaped coined has a greater chance of landings heads than landing tails.
10. The Houston Astros have a 22% chance of winning the 2018 World Series.

We apply the same three-step process to these sorts of *contextual* chances. *Step #1:* We associate the assertion with some partition, picked out by the context. The elements of the partition are the chancemakers for the kind of contextual chance being invoked. *Step #2:* We claim that propositions concerning the kind of contextual chance being invoked are *a priori* equivalent to the disjunction of their possible chancemakers. *Step #3*: We claim that the epistemic theory of chancemaking holds of the kind of contextual chance at issue. Again, these claims ensure that the sort of contextual chance at issue will constrain credence in the same, special way that fundamental chance does.

Different contexts pick out different partitions. In a scientific context, the relevant partition for an utterance of (9) might be $\boldsymbol{LH_t}$. In an ordinary context, however, the relevant partition will be coarser-grained, perhaps being composed of propositions that specify what the available coin-relevant information might be. In a similar way, the relevant partition for an utterance of (10) might be $\boldsymbol{B}$, a partition whose elements specify what the available baseball-relevant information might be.

Thinking about the partitions associated with contextual chances helps shed light on why chance-talk is so useful. If the epistemic theory of chancemaking is true, then we could dispense with chance-talk altogether. Every chance proposition is *a priori* equivalent to a claim that nowhere mentions chance: namely, the disjunction of its chancemakers. An ordinary assertion of (34), for example, is *a priori* equivalent to $B_1 \vee B_2 \vee ...$, a disjunction of the aforementioned partition of available baseball-relevant information. If we had some easy way of asserting this disjunction, we could assert it and leave chance unmentioned. But we don't have any easy way to assert $B_1 \vee B_2 \vee ...$. Asserting even one of its disjuncts would require thousands, maybe tens or hundreds of thousands, of words. And it is for precisely that reason that chance-talk is so useful. It allows us to quickly summarize our epistemic situation.

Think about an example. Your friend wants to know how confident she should be that the Houston Astros will win the 2018 World Series. You are a baseball nut, a Sabermetrician, who knows all of the available baseball-relevant information, but it would take you a week to communicate to your friend what you know. The best, most efficient way to get your friend what she wants is to speak in terms of chance and say, “The Houston Astros have a 22% chance of winning the 2018 World Series.” You thereby give your friend what she wants—you communicate that the available baseball-relevant information is such that every rational prior, when updated on it, is 22% confident that the Houston Astros will win the 2018 World Series—without having to get into the mind-bogglingly complicated details of what the available baseball-relevant information in fact is.[20]

§5/ Humean Supervenience and Superchance

In sections 2-4 we assumed the falsity of Humean Supervenience and argued that non-Humeans, like us, should accept the epistemic theory of chancemaking. We now consider chance from a Humean point of view and argue that, while Humeans should reject the epistemic theory of chancemaking, they should accept the epistemic theory of superchancemaking.

The first claim is straightforward. Take Initial Principle (a), which, when applied to fundamental chance, states that for any rational prior $C_0(\text{-})$, $C_0(P|\langle \mathrm{Ch}_{F,t}(P) = x\rangle) = x$, if defined. As it turns out, this claim, although entailed by the epistemic theory of chancemaking, is incompatible with Humean Supervenience.

[20] One might worry that the various different kinds of chance would imposed incompatible constrains on rational credence; *cf.* Meacham (2014). But the epistemic theory of chancemaking ensures that different kinds of chance will not impose incompatible constraints on rational credence.

The incompatibility—sometimes called the problem of undermining futures, sometimes called the big bad bug—is easy to see.[21] The chances allow total courses of history that are misleading indicators of what the chances are. Take tritium decay. The half-life of tritium is 12.26 years, so each newly created tritium atom has a 0.5 chance of decaying in that timespan. Some tritium atoms might decay very quickly, in under three minutes, say. And although the chance is very small indeed, there is a nonzero chance that every future tritium atom will decay very quickly. Let T be a description of the total course of history, on which there are many more tritium atoms in the future than there have been up to now, and on which every future tritium atom decays in under three minutes. The t-chance of T is $z > 0$. So, according to Initial Principle (a), if $C_0(\text{-})$ is a rational prior, $C_0(T|\langle Ch_{F,t}(T) = z\rangle) = z > 0$. But, if Humean Supervenience is true, then $C_0(T|\langle Ch_{F,t}(T) = z\rangle) = 0 < z$. According to Humeans, the chances supervene on the total course of history, and they do so in a way that ensures that the total course of history is not a misleading indicator of what the chances are. According to Humeans, the half-life of tritium is the same at every T-world, and the half-life of tritium at T-worlds is much less than 12.26 years. Thus, according to Humeans, T and $\langle Ch_{F,t}(T) = z\rangle$ are *a priori* incompatible. The t-chance of T is z only at world at which the half-life of tritium is 12.26 years, but we can know *a priori* that there are no T-worlds at which the half-life of tritium is 12.26 years. And since every rational prior assigns credence one to every *a priori* truth, if $C_0(\text{-})$ is a rational prior, then $C_0(T|\langle Ch_{F,t}(T) = z\rangle) = 0 < z$.

---

[21] See *e.g.* Artnzenius and Hall (2003), Bigelow, Collins, and Pargetter (1993), Briggs (2009b), Hall (1994; 2004), Hoefer (2007), Ismael (2008), Lewis (1994), and Schaffer (2003).

The incompatibility has nothing especially to do with Initial Principle (a) or fundamental chance. For just about any kind of chance, Humean Supervenience is incompatible with the Initial Principles, the Evidential Principles, and the Resiliency Principles as applied to it. Humean Supervenience, to put the point tersely, is incompatible with the claim that chance constrains credence.

Chance *seems* to constrain credence, of course, so Humeans, in an effort to save appearances, are quick to point out that their view, although incompatible with with the claim that chance constrains credence, is compatible with the claim that *superchance*, a close cousin of chance, constrains credence.[22]

If *K*-chance is some kind of chance, then the *K-superchances* at some world *w* are the *K*-chances at *w* conditional on the *K*-chancemaker at *w*. For example, the *t*-superchance of *P* at world *w* is the *t*-chance of *P* conditional on LH_{tw}.[23]

For non-Humeans, the chances are always already conditioned on their chancemakers, so there is no distinction between chance and superchance.

For Humeans, however, chance and superchance are importantly different. The superchances do not allow total courses of history that are misleading indicators of what the

[22] See *e.g.* Hall (1994) and Lewis (1994).

[23] The *t*-superchances are thus the *t*-chances conditioned on what Hall (1994: 511) calls the "complete history-law conjunction." Also see Lewis (1994: 487).

superchances are. (If Humean Supervenience holds, then the *t*-superchance of *T* is zero.) Thus, Humeans can and do claim that superchance constrains credence.[24]

To say that superchance constrains credence is to say that various superchance-credence norms are true. If *K*-superchance is some kind of superchance, then there will be various *K-superchance propositions.* Among them:

- $\langle \mathrm{Sch}_K(P) = x \rangle$: the proposition that the *K*-superchance of *P* equals *x*.
- $\langle \mathrm{Sch}_K(P) \approx x \rangle$: the proposition that the *K*-superchance of *P* approximates *x*.
- $\langle \mathrm{Sch}_K(P) < \mathrm{Sch}_K(P|Q) \rangle$: the proposition that the *K*-superchance of *P* is less than the *K*-superchance of *P* conditional on *Q*.
- $\langle \mathrm{Sch}_K(P) = (2)(\mathrm{Sch}_K(Q)) \rangle$: the proposition that the *K*-superchance of *P* is twice the *K*-superchance of *Q*.

There also will be various *superchance-credence norms* concerning *K*-superchance: namely, the Initial Principles, the Evidential Principles, and the Resiliency Principles. And Humeans accept all of these superchance-credence norms: For every chance-credence norm that non-Humeans accept, Humeans accept the corresponding superchance-credence norm.

Because Humeans think that superchance constrains credence, they need to explain why superchance constrains credence. Which brings us to our second claim: Humeans should accept *the epistemic theory of superchancemaking*.

---

[24] They do so by rejecting the Principal Principle in favor of the New Principle. See *e.g.* Arntzenius and Hall (2003), Briggs (2009a), Hall (1994; 2004) and Lewis (2004).

According to the epistemic theory of superchancemaking, if *C* is some superchance proposition and *M* is some nonsuperchance proposition,

> If *M* is a possible superchancemaker for *C*, then it is so because every rational prior, when updated on *M*, reflects *C*.[25]

Given the epistemic theory of superchancemaking and the claim that every superchance proposition is *a priori* equivalent to the disjunction of its possible superchancemakers, every true superchance-credence norm can be shown to follow from the probability axioms. The proofs parallel the ones above.

Looking at superchance reveals how flexible the epistemic theory is. In principle, it could be used to explain why any objective probability function constrains credence. If you think that schmances are the objective probabilities that constrain credence, you should accept the epistemic theory of schmancemaking. If, like Humeans, you think that the objective probabilities that constrain credence are superchances, you should accept the epistemic theory of superchancemaking. If, like non-Humeans, you think that the objective probabilities that constrain credence are chances, you should accept the epistemic theory of chancemaking.

This essay is not the place to decide the dispute between non-Humeans and Humeans, but it is worth making one point in favor of non-Humeans. Scientists are interested in chance and not interested in superchance. Chances feature in laws of nature and scientific explanations. Billions of dollars are spent trying to learn about the chances. By contrast, scientists have never even

[25] For any *K* and any world *w*, the *K*-chancemaker at *w* will also be the *K*-superchancemaker at *w*.

heard of superchance. Thousands of years of ever-improving science and, so far as we call tell, no scientist has ever had the need to say anything about superchance. There would be no inconsistency if Humeans held tight to their metaphysical commitments and insisted that scientists are simply uninterested in the objective probabilities that are epistemically important, the ones that constrains credence. But such a view is clearly odd. Why should one sort of objective probability be important for metaphysics and science, and another sort be important for epistemology?

Those of us who are more deferential to science will prefer the non-Humean view, according to which the objective probabilities that are epistemically important are the very ones that scientists are in the business of investigating. It would take a very powerful argument indeed to convince us that the objective probabilities that constrain credence are not the chances.

§6/ The Question of Rational Priors

Having completed our expositions of the epistemic theory of chancemaking, we now turn to drawing out some of its consequences for the question of which priors are rational. There is a spectrum of view about which priors are rational. At one extreme are Bayesian subjectivists, who hold that every probability function is a rational prior. At the other extreme are Bayesian objectivists, who hold that only one probability function is a rational prior. The epistemic theory of chancemaking puts pressure on views toward the subjectivist end of the spectrum.

Take subjectivism, itself. According to subjectivism, if *P* and *Q* are logically independent, a rational prior can assign any probability to *P* conditional on *Q*. Given the epistemic theory of chancemaking, subjectivism entails that nontrivial chance is impossible. To see this, consider some nontrivial chance proposition, ⟨$\text{Ch}_K(P) = x$⟩, which ascribes to *P* some chance other than zero, one, or the entire unit interval.[26] If ⟨$\text{Ch}_K(P) = x$⟩ is true at some world *w*, then, according to the epistemic theory, there will be some proposition, *Q*, that is logically independent of *P* and the chancemaker of ⟨$\text{Ch}_K(P) = x$⟩ at *w*. If *Q* is a possible chancemaker of ⟨$\text{Ch}_K(P) = x$⟩, then, according to the epistemic theory of chancemaking, every rational prior assigns *x* probability to *P* conditional on *Q*. But this contradicts subjectivism, which says that a rational prior can assign any probability to *P* conditional on *Q*.

The epistemic theory of chancemaking thus gives us a powerful argument against subjectivism. Given the epistemic theory of chancemaking, subjectivism is refuted by the mere possibility of nontrivial chance.

An exactly similar argument can be run against a moderate view near the subjectivist end of the spectrum, which we call the *chance-moderate view*. According to the chance-moderate view, verifying the chance-credence norms is the only constraint on being a rational prior; if *P* and *Q* are nonchancy propositions that are logically independent, then a rational prior can assign any probability to *P* conditional on *Q*.

The chance-moderate view seems to be an attractive view that strikes a principled balance between subjectivism and objectivism. (From personal experience, we know a number of philosophers sympathetic to it.) But if the epistemic theory of chancemaking is true, then the chance-moderate view does not lie between subjectivism and objectivism; rather, it is equivalent to subjectivism. According to the epistemic theory of chancemaking, chance-credence norms are, absent other rational constraints, vacuous and verified by every probability function. Thus, if

[26] Subjectivism is compatible with a propositions having trivial chances.

verifying chance-credence norms is the only requirement on being a rational prior, as the proponent of the chance-moderate view asserts, every probability function is a rational prior.

As we already saw, given the epistemic theory of chancemaking, the claim that every probability function is a rational prior entails that nontrivial chance is impossible. We thus have a new and powerful argument against the chance-moderate view. Given the epistemic theory of chancemaking, the chance-moderate view, like subjectivism, is refuted by the mere possibility of nontrivial chance.

Other moderate positions toward the subjectivist end of the spectrum will be compatible with the possibility of nontrivial chance but will be incompatible with the claims about nontrivial chance that scientists and other experts actually make. Take a societal example: According to NPR (National Public Radio), there is a 0.401 chance that judges will be automated by 2035.[27] (By contrast, the chance that roofers will be automated by 2035 is 0.897.) Call the evidence that NPR appeals to *E*. If NPR is right—if the chance that judges will be automated by 2035 is 0.401—then *every* rational prior is 0.401 confident that judges will be automated by 2035 conditional on *E*. Moderates toward the objectivist end of the spectrum will be happy to take this in stride. Already convinced that there is widespread agreement among rational priors, they won't balk at the claim that every rational prior is 0.401 confident that judges will be automated by 2035 (and 0.897 confident that roofers will be automated by 2035) conditional on *E*. But moderates toward the subjectivism end of the spectrum are forced to be error theorists. If they want to maintain that *some* rational prior could be 0.399 confident that judges will be automated by 2035 conditional on *E*, then they forced to deny the claims about chance made by a reputable

[27] https://www.npr.org/sections/money/2015/05/21/408234543/will-your-job-be-done-by-a-machine

sources. The only moderate positions that can accomodate our ordinary chance-talk are moderate positions fairly far toward the objectivist end of the spectrum.

Finally, consider objectivism. A benefit of objectivism is that it accommodates our ordinary chance-talk, allowing chance-facts about coins, dies, weather, sports, politics, biology, physics, and future automation. But one might worry that objectivism is *too* strong, since, given the epistemic theory of chancemaking, it predicts that *any* proposition has a precise chance relative to *any* contextually salient partition of possible chancemakers.

Does this implication of objectivism fly in the face of our ordinary chance-talk? We are not sure. If objectivism was false, then there should be cases in which asking about the precise chance of a proposition, relative to some contextually salient partition of chancemakers, involves some sort of presupposition failure. But we do not find these sorts of presuppositions failures. We say:

11. I have no idea what my chances are that I'll get a tenure-track job.
12. Who knows what the chance is that North Korea will be denuclearized by 2030?

But we do not say:

13. There is no fact of the matter as to what the chance is that I'll get a tenure-track job.
14. Denuclearization is not the sort thing that could have a chance.

So far as we can tell, nothing in our ordinary chance-talk conflicts with objectivism.

In sum, the epistemic theory of chancemaking pushes us towards the objectivist end of the spectrum on the question of which priors are rational. Views towards the subjectivist end of the spectrum are committed to a widespread error theory about much of our chance discourse.

§7/ The Chance Is-Ought Gap(s)

We suspect that some readers will feel that we have sidestepped the most important epistemological problem about chance. Once we take on board the epistemic theory of chancemaking, the obvious follow-up question is: *Why?* Supposing that M is a possible chancemaker for some chance proposition C, *why* does every rational prior, when updated on M, reflect C? Why, for example, does every rational prior assigns 0.401 probability to the proposition that judges will be automated b y 2035 conditional on certain facts about technology and the ins and outs of being a judge? How do we move the "is-fact" that M is true to the epistemic "ought-fact" that every rational prior reflects C conditional on M? Call this the *problem of the chance is-ought gap.*

We think that the problem of the chance is-ought gap is a hopelessly disunified and gerrymandered problem, and there is no reason to expect a unified solution to it precisely *because* it is so disunified and gerrymandered. That's why we have not addressed the chance is-ought gap in this paper. As we have stressed, chance-talk appears in many, many domains of inquiry. It stands to reason that the explanation for why every rational prior reflects C conditional on M depends on the details of M and C.

Take fundamental chance, for example. Suppose that LH_t makes the t-chance of P equal x. One then might ask, why does every rational prior assign x probability to P conditional LH_t?

We think the answer to this question depends on the metaphysics of laws. If a Humean conception of laws is true, then laws are simple, compact summaries of frequencies. Thus, if there is any explanation for why every rational prior assigns *x* probability to *P* conditional on LH_t, it will appeal to some fact about the rational response to the sort of frequency data encoded by LH_t. If laws are second-order universals,[28] then the explanation will instead appeal to the rational response to certain relations among universals.[29] If a primitivist account of laws is true,[30] then the explanation, if there is one, will appeal to some fact about the rational response to certain sorts of metaphysical primitives.

When we turn to nonfundamental chances, the explanations get more complex and more diverse. There are some relatively simple cases, like coins and dies, in which the explanation appears to invoke the principle of indifference. When symmetries are the possible chancemakers, it is plausible that we can appeal to the principle of indifference to explain why claims about symmetries constrain rational credence as they do. Similarly, by applying the principle indifference to the micro-states that can realize macro-states, we can perhaps explain why the chancemakers for statistical mechanical chance constrain credence as they do.[31] But consider the unsurveyably complex fact, *M*, that makes it true that the Houston Astros have a 22% chance of winning the 2018 World Series. No application of the principle of indifference will explain why every rational prior is 0.22 confident that the Houston Astros will win the 2018 World Series

---

[28] *Cf.* Dretske (1977), Tooley (1977), and Armstrong (1983).

[29] This question is closely related to van Fraassen's (1989) inference problem.

[30] See *e.g.* Maudlin (2007) and Carroll (1994).

[31] See *e.g.* Meacham (2010b).

conditional on *M*. The explanation, if there is one, will be very complicated indeed, and very different from the ones above.

More or less *any* way that one proposition can evidentially support another is a possible way that some possible chancemaker constrains rational credence as it does. A solution to the problem of the chance is-ought gap would require a full and complete account of how propositions can evidentially support one another. Of course, an account of how propositions can evidentially support one another would be nice. But we can fully explain why chance constrains credence without it.

§8/ Conclusion

The epistemic theory of chancemaking leaves undecided many of the main questions in the metaphysics of chance. For example, it does not tell us whether the chancemakers for fundamental physical chances involve any non-Humean laws, powers, or propensities. The epistemic theory of chancemaking also leaves open how widespread and determinate the chances are. This question is closely related to the question of which priors are rational. As it turns out, if we want to explain why chance constrains credence, these questions do not need to be settled.

One might have thought that chance played some deep and ineliminable role in the theory of rational credence. But if the epistemic theory of chancemaking that we have defended is true, chance-credence norms are theorems of the probability calculus. Chance does not place any additional constrains on rational credence; it merely serves to summarize the independent, nonchancy rational constraints. The whole truth about rational credence could thus be told without ever mentioning chance.

References

Armstrong, David. 1983. *What Is a Law of Nature?* Cambridge: Cambridge University Press.

------. 2004. *Truth and Truthmakers*. Cambridge University Press.

Arntzenius, Frank and Ned Hall. 2003. On What We Know About Chance. *British Journal for the Philosophy of Science* 54: 171-9.

Bigelow, John. 1988. *The Reality of Numbers: A Physicalist's Philosophy of Mathematics.* Oxford University Press.

Bigelow, John, John Collins, and Robert Pargetter. 1993. The Big Bad Bug: What Are the Humean's Chances? *British Journal for the Philosophy of Science* 44: 443-62.

Bigelow, John and Robert Pargetter. 1990. *Science and Necessity*. Cambridge University Press.

Black, Robert. 1998. Chance, Credence, and the Principal Principle. *British Journal for the Philosophy of Science* 49: 371-85.

Briggs, R. A. 2009a. Distorted Reflection. *Philosophical Review* 118: 59–85.

------. 2009b. The Anatomy of the Big Bad Bug. *Noûs* 43, no. 3: 428–449.

Carroll, John. 1994, *Laws of Nature*, Cambridge University Press.

Dretske, Fred. 1977. Laws of Nature. *Philosophy of Science*, 44: 248–268.

Elliott, Katrina. 2016. Explaining (One Aspect of) the Principal Principle without (Much) Metaphysics. *Philosophy of Science* 83: 480-99.

Emery, Nina. 2017. A Naturalist's Guide to Objective Chance. *Philosophy of Science* 84: 480-99.

van Fraassen, Bas. 1984. Belief and the Will. *Journal of Philosophy* 81: 235-56.

------. 1989. *Laws and Symmetry*, Clarendon Press.

Gallow, Dmitri J. MS. A Subjectivist's Guide to Deterministic Chance.

Hall, Ned. 1994. Correcting the Guide to Objective Chance. *Mind* 103: 505-18.

------. 2004. Two Mistakes About Credence and Chance. *Australasian Journal of Philosophy* 82: 93-111.

Handfield, Toby and Alastair Wilson. 2014. Chance and Context. In Wilson 2014, pp. 19-44.

Hoefer, Carl. 2007. The Third Way on Objective Probability: A Sceptic's Guide to Objective Chance. *Mind* 116: 549–596.

Howson, Colin and Peter Urbach. 1989. *Scientific Reasoning: The Bayesian Approach*. Open Court Publishing Company.

Ismael, Jenann. 2008. Raid! Dissolving the Big, Bad Bug. *Noûs* 42: 292-307.

Lange, Marc. 2017. Is Kantian Projectivism the Only Hope for Grounding to Principal Principle? (No.). *Monist* 100: 422-36.

Lewis, David. 1980. A Subjectivist's Guide to Objective Chance. In *Studies in Inductive Logic and Probability*, edited by Richard C. Jeffrey, 83–132. University of California Press.

------. 1994. Humean Supervenience Debugged. *Mind* 103: 473–490.

Littlejohn, Clayton. 2018. Stop Making Sense? On a Puzzle About Rationality. *Philosophy and Phenomenological Research* 96: 257–272.

Loewer, Barry. 2001. Determinism and Chance. *Studies in History and Philosophy of Science Part B*. 32: 609-20.

Mackie, J. L. 1977. *Ethics: Inventing Right and Wrong*. Penguin Books.

Maudlin, Tim. 2007. *The Metaphysics Within Physics*. New York: Oxford University Press.

Meacham, Christopher. 2005. Three Proposals Regarding a Theory of Chance. *Philosophical Perspectives* 19: 281–307.

------. 2010a. Two Mistakes Regarding the Principal Principle. *British Journal for the Philosophy of Science* 61: 407–431.

------. 2010b. Contemporary Approaches to Statistical Mechanical Probabilities: A Critical Commentary - Part I: The Indifference Approach. *Philosophy Compass* 5: 1116–1126.

------. 2014. Autonomous Chances and the Conflicts Problem. In Wilson 2014, 45-67.

Mellor, D. H. 1971. *The Matter of Chance*. Cambridge University Press.

Pettigrew, Richard. 2012. Accuracy, Chance, and the Principal Principle. *Philosophical Review* 121: 241-75.

------. 2013. A New Epistemic Utility Argument for the Principal Principle. *Episteme* 10: 19-35.

------. 2016. *Accuracy and the Laws of Credence*. Oxford University Press.

Popper, Karl. 1982. *Quantum Theory and the Schism in Physics*. Rowman and Littlefield.

Rayo, Agustín. MS. An Anti-Reductionist's Guide to Evidential Support.

Schaffer, Jonathan. 2003. Principled Chances. *British Journal for the Philosophy of Science* 54: 27-41.

------. 2007. Deterministic Chance? *British Journal for the Philosophy of Science* 58: 113-40.

Schwarz, Wolfgang. 2014. Proving the Principal Principle. In Wilson (ed.), *Chance and Temporal Asymmetry*, pp. 81-99. Oxford University Press.

Skyrms, Brian. 1977. Resiliency, Propensities, and Causal Necessity. *Journal of Philosophy* 74: 704-13.

Strevens, Michael. 1998. Inferring Probabilities From Symmetries. *Noûs* 32: 231–246.

------. 1999. Objective Probability as a Guide to the World. *Philosophical Studies* 95: 243–275.

Titelbaum, Michael. 2015. Rationality's Fixed Point (Or: In Defense of Right Reason). In T. Gendler and J. Hawthorne (eds.) *Oxford Studies in Epistemology* 5. Oxford University Press.

Tooley, Michael. 1977. The Nature of Laws. *Canadian Journal of Philosophy* 7: 667–698.

Wilson, Alistair (ed.). 2014. *Chance and Temporal Asymmetry*. Oxford University Press.

Weisberg, Jonathan. 2007. Conditionalization, Reflection, and Self-Knowledge. *Philosophical Studies* 135: 179-97.