

## *No Crystal Balls*

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### 1. Introduction

A crystal ball, in Hall's (1994) sense, need be neither round nor crystalline. The world is said to contain crystal balls whenever the present carries news of the as-yet-undetermined parts of the future. Images appearing in spheres made of magical quartz might be crystal balls, in the relevant sense, but so too might arrangements of magical tealeaves or neural states in the brains of time travelers or clairvoyants.

Many philosophers believe that crystal balls are metaphysically possible.<sup>1</sup> In this essay, I argue that they are not.

Whether crystal balls are possible matters, for at least two reasons. The first is epistemological. According to a simple, user-friendly chance norm for credence, which I call the *Present Principle*, agents are rationally required to conform their credences to their expectations of the present chances, deferring to the present chances as they would to an expert. I would like to defend the Present Principle since its truth would do much to simplify the relation between chance and credence. But the Present Principle is counterexample-free, and hence defensible, only if crystal balls are impossible.

The second reason is decision-theoretic. The problem of crystal balls is one of the main objections to causal decision theory.<sup>2</sup> Critics of causal decision theory, Egan (2007) and Price (2012), claim that crystal ball cases are counterexamples to causal decision theory. But even proponents worry. Lewis, for example, in his 1981 defense of causal decision theory, draws special attention to the problem. He says that crystal ball cases are “much more problematic for decision theory than the Newcomb problems” (1981, 18) and then sets such cases aside, restricting his defense of causal decision theory.

I believe that Lewis's restriction is unnecessary and that the problem of crystal balls can be laid to rest. Crystal ball cases can be counterexample to causal decision theory only if crystal balls are possible, and, as I argue below, crystal balls are not possible.

### 2. Credence Norms

I assume that each perfectly rational agent begins her life with a reasonable initial credence function. To make things simpler, I assume that there is only one reasonable initial credence function, namely,  $C_0(\cdot)$ .

At any non-initial time, a perfectly rational agent has a non-initial credence function. Let  $C_t(\cdot)$  be the agent's non-initial credence function at time  $t$ . Initial and non-initial credence functions are connected by the following evidential norm:

**The Total Evidence Principle.** If  $E$  is an agent's (total) evidence at  $t$ , then rationality requires that  $C_t(\cdot) = C_0(\cdot | E)$ .

Evidence thus plays the *certainty-licensing role*: a perfectly rational agent is certain, and willing to bet at any odds, on the truth of any proposition entailed by her evidence.

There are also chance norms for credence, of which the most famous is Lewis's (1980) Principal Principle. Let  $t$  be any time. Let  $x$  be any real number on the unit interval. Let  $\langle \text{Ch}_t(P) = x \rangle$  be a  $t$ -chance proposition that is true just if the chance, at time  $t$ , of  $P$  equals  $x$ . Let  $Q$  be any proposition that is compatible with  $\langle \text{Ch}_t(P) = x \rangle$ . Then we have

**The Principal Principle.** If  $Q$  is admissible with respect to  $\langle \text{Ch}_t(P) = x \rangle$ , then  $C_0(P|Q \& \langle \text{Ch}_t(P) = x \rangle) = x$ .

Think of the Principal Principle as an attempt to solve a goldilocks problem. On the one hand, there are chance norms for credence that are true but too weak, for example,

**The Initial Principle.**  $C_0(P | \langle \text{Ch}_t(P) = x \rangle) = x$ .

Although true, the Initial Principle says nothing about non-initial credences and is therefore too weak.

On the other hand, there are chance norms for credence that are plenty strong but false, for example,

**The Unrestricted Principle.**  $C_0(P|Q \& \langle \text{Ch}_t(P) = x \rangle) = x$ .

The Unrestricted Principle is false. Let  $x$  be less than 1, and let  $Q$  be  $P$ . Then, according to the Unrestricted Principle,  $C_0(P|P \& \langle \text{Ch}_t(P) = x \rangle) = x$ , whereas, in truth,  $C_0(P|P \& \langle \text{Ch}_t(P) = x \rangle) = 1$ .

The Principal Principle solves the goldilocks problem by allowing us to strengthen the Initial Principle cautiously. We first define *admissible* propositions as those that leave unaffected the agent's rightful deference to the chances:

$Q$  is *admissible* with respect to  $\langle \text{Ch}_t(P) = x \rangle$  if and only if  $Q$  is compatible with  $\langle \text{Ch}_t(P) = x \rangle$  and  $C_0(P|Q \& \langle \text{Ch}_t(P) = x \rangle) = x$ , and  $Q$  is *inadmissible* with respect to  $\langle \text{Ch}_t(P) = x \rangle$ , otherwise.<sup>3</sup>

We then can defend gradually stronger theses about which propositions are admissible with respect to which others. If no proposition is admissible with respect to any other, the Principal Principle is weak and equivalent to the Initial Principle. If every proposition is admissible with respect to every other, the Principal Principle is false and equivalent to the Unrestricted Principle. But there are many strong yet true claims that lie between the Initial Principle and the Unrestricted Principle, and

the cardinal virtue of the Principal Principle is that it allows us to state and defend these intermediate positions.

Although formulated in terms of conditional credences, the Principal Principle induces a constraint on unconditional credences. Let  $eCh_t(P)$  be the agent's expectation, at time  $t$ , of the  $t$ -chance of  $P$ —that is,  $\sum_x C_t((Ch_t(P) = x))x$ . If the agent's evidence at  $t$  is admissible with respect to every  $\langle Ch_t(P) = x \rangle$  with which it is compatible, then the Principal Principle entails that  $C_t(P)$  should equal  $eCh_t(P)$ —that, in other words, the agent's credence in  $P$  should conform to her expectation of the  $t$ -chance of  $P$ .

### 3. The Present Principle

Like the Principal Principle, the Present Principle is a proposed chance norm for credence. If  $E$  is an agent's evidence at time  $t$ , and  $E$  is compatible with  $\langle Ch_t(P) = x \rangle$ , then we have

**The Present Principle.**  $C_0(P|E \& \langle Ch_t(P) = x \rangle) = x$ .

The Present Principle nowhere mentions admissibility, which is a significant, albeit risky, virtue.

The admissibility clause insulates the Principal Principle from refutation. So long as the Initial Principle is true, the Principal Principle is true, since we may take the Principal Principle to be the strongest true strengthening of the Initial Principle. But the insulation comes at the cost of decreased utility. Determining which propositions are admissible with respect to which others is a deep and difficult philosophical matter. (The Principal Principle has been around for almost 40 years, and still there is no philosophical consensus!)<sup>4</sup> To use the Principal Principle, therefore, agents must wade into deep and difficult philosophical waters, somehow determining whether their evidence is admissible with respect to the relevant chance propositions. If agents are uncertain what their evidence is or whether they are perfectly rational, the Principal Principle may be prohibitively difficult to use.

By contrast, the Present Principle is very easy to use. There is no need to wade into deep and difficult philosophical waters. Agents can use the Present Principle without having to settle any questions about admissibility.

But the user-friendliness of the Present Principle comes at the cost of increased exposure to refutation. Say that an agent's evidence at  $t$  is *presently inadmissible* just if the agent's evidence is inadmissible with respect to some  $t$ -chance proposition with which it is compatible. If it is *possible* for an agent's evidence to be presently inadmissible, then the Present Principle is false. If  $E$  is an agent's evidence at  $t$ , and  $E$  is inadmissible with respect to some  $\langle Ch_t(P) = x \rangle$  with which it is compatible, then by the definition of inadmissibility,  $C_0(P|E \& \langle Ch_t(P) = x \rangle) \neq x$ , whereas the Present Principle entails that  $C_0(P|E \& \langle Ch_t(P) = x \rangle) = x$ . On the other hand, if it is *impossible* for an agent's evidence to be presently inadmissible, then the Present Principle is true and, in fact, follows from the Principal Principle.

#### 4. The Past Principle

It is worth comparing the Present Principle with a principle that requires agents to defer to the *past* chances. Let  $t-$  be some time before  $t$ . Let E be an agent's evidence at  $t$ , and suppose that E is compatible with  $\langle \text{Ch}_t(\text{P}) = x \rangle$ . Then we have

**The Past Principle.**  $C_0(\text{P}|\text{E} \& \langle \text{Ch}_t(\text{P}) = x \rangle) = x$ .

Everyone agrees that the Past Principle is false. Counterexamples abound.

Take a simple illustration. At time  $t-$ , an oddly shaped coin is tossed into the air. Afterwards, at time  $t$ , the agent sees the coin land heads. Suppose that the agent then learns that the chance, at  $t-$ , of the oddly shaped coin landing heads was 0.5. If the Past Principle were true, the agent, upon learning this fact about the past chances, would be rationally required to become 0.5 confident that the oddly shaped coin landed heads. But clearly that's not so. The agent should remain (virtually) certain that the coin landed heads.

The failure of the Past Principle is easy to explain. Unlike the agent, the chances, at time  $t-$ , have not "seen" the outcome of the coin toss and do not "know" how the oddly shaped coin landed. In general, it is not rational for an agent to defer to a putative expert if the agent has reason to believe that she possesses information not possessed by the putative expert, and an agent in the present almost always has reason to believe that she possesses information that is not "possessed" by the past chances. An agent in the present knows about the present, after all, whereas the past chances do not.

#### 5. An Alleged Counterexample to the Present Principle

If crystal balls are possible, then there will be circumstances in which an agent in the present has reason to believe that she possesses information that is not "possessed" by the *present* chances. The Present Principle will then fail for the same reason that the Past Principle fails. An agent in the present will know about the future, whereas the present chances will not.

Cases such as the following have convinced many philosophers that crystal balls *are* possible:

*Magical quartz.* Ann knows that an oddly shaped coin will be tossed tomorrow. Having tossed this particular coin many times before and studied its geometric properties, she knows that the present chance of the coin landing heads is 0.5. As it happens, Ann is looking into a crystal ball—literally, a sphere made of magical quartz. Ann, who has excellent reason to trust the prophetic powers of the crystal ball into which she is looking, sees an image of the oddly shaped coin being tossed tomorrow and landing heads.

If *Magical Quartz* is coherently described, then Ann's sphere of magical quartz is a crystal ball, in Hall's sense, and the Present Principle stands refuted. Let E be Ann's evidence at  $t$ , and let  $\langle \text{Heads} \rangle$  be the proposition that the oddly shaped coin lands heads. According to the details of the case, E entails  $\langle \text{Ch}_t(\langle \text{Heads} \rangle) = 0.5 \rangle$ . So, assuming that Ann is perfectly rational,  $C_t(\langle \text{Heads} \rangle) =$

$C_0(\langle \text{Heads} \rangle | E) = C_0(\langle \text{Heads} \rangle | E \& \langle \text{Ch}_t(\langle \text{Heads} \rangle) = 0.5 \rangle)$ . The Present Principle entails that, deferring to the present chances, Ann should be 0.5 confident in  $\langle \text{Heads} \rangle$ . But that seems clearly false. It seems that Ann should be much more than 0.5 confident in  $\langle \text{Heads} \rangle$ .

We thus have one putative crystal ball (Ann's sphere) and one putative counterexample to the Present Principle (*Magical Quartz*).

## 6. The Rule of U-maximization

Counterexamples to the Present Principle can be transformed into counterexamples to causal decision theory.

In the usual way, let us model an agent facing a decision as a quadruple,  $\langle \mathbf{A}, \mathbf{O}, C_t(\cdot), u(\cdot) \rangle$ . The first coordinate,  $\mathbf{A}$ , is the set of *options*, the prospects-cum-propositions among which the agent is choosing. The second coordinate,  $\mathbf{O}$ , is the set of possible *outcomes*, the propositions that are the fundamental bearers of noninstrumental desire, the ends for which options are chosen as means. The third coordinate,  $C_t(\cdot)$ , is the agent's credence function at the time of decision,  $t$ . The fourth coordinate,  $u(\cdot)$ , is the agent's *utility function*.

There are various ways to formulate causal decision theory, but, like Skyrms (1980; 1984) and Lewis (1981), I think that the most plausible formulations are those in terms of chance.<sup>5</sup> If we assume that agents are always rationally certain that each of their options has a nonzero present chance of being chosen (as I will, hereafter), then we can formulate causal decision theory in terms of conditional chance.<sup>6</sup> For any option  $A$  and for any outcome  $O$ , there is a  $t$ -chance proposition,  $\langle \text{Ch}_t(O|A) = x \rangle$ , that is true just if the  $t$ -chance of  $O$  conditional on  $A$  equals  $x$ . Let  $e\text{Ch}_t(O|A)$  be the agent's expectation, at time  $t$ , of the  $t$ -chance of  $O$  conditional on  $A$ —that is,  $\sum_x C_t(\langle \text{Ch}_t(O|A) = x \rangle)x$ . The U-value of option  $A$  then equals  $\sum_O e\text{Ch}_t(O|A)u(O)$ , and we arrive at the decision rule at the heart of causal decision theory:

**The Rule of U-maximization.** Rationality requires that agents choose so as to maximize U.

It is easy to see why counterexamples to the Present Principle can be transformed into counterexamples to the Rule of U-maximization. If the Present Principle admits of counterexamples, then a rational agent's credences diverge from her expectations of the present chances. In decision theory, then, when it comes to weighing the utilities of the possible outcomes, a question arises whether to use the credences or the expectations of the present chances as the relevant weights. The Rule of U-maximization entails that agents should use their expectations of the present chances, but upon considering examples, it seems clear that agents should use their credences.

Take *Magical Quartz*. Assuming that the example is coherently described, Ann's evidence at  $t$  entails  $\langle \text{Ch}_t(\langle \text{Heads} \rangle) = 0.5 \rangle$ , so  $e\text{Ch}_t(\langle \text{Heads} \rangle) = 0.5$ . But  $C_t(\langle \text{Heads} \rangle) > 0.5$ : Ann, as she should be, is much more than 0.5 confident that the oddly shaped coin will land heads. Now suppose that there is a bet that wins \$2 if

the oddly shaped coin lands heads and loses \$3 if the coin lands tails, and suppose that Ann has two options: she can take the bet ( $A_{\text{BET}}$ ), or refuse it ( $A_{\text{REFUSE}}$ ). The Rule of U-maximization recommends that Ann refuse the bet, as we can see by running through the calculations:

Let  $O_2$ ,  $O_0$ , and  $O_{-3}$  be the outcomes of winning \$2, breaking even, and losing \$3, respectively. Then, equating dollars and utility:

$$U(A_{\text{REFUSE}}) = \sum_O e\text{Ch}_t(O|A_{\text{REFUSE}}) u(O) = e\text{Ch}_t(O_0|A_{\text{REFUSE}})(0) = (1)(0) = 0$$

$$U(A_{\text{BET}}) = \sum_O e\text{Ch}_t(O|A_{\text{BET}}) u(O) = e\text{Ch}_t(O_2|A_{\text{BET}})(2) - e\text{Ch}_t(O_{-3}|A_{\text{BET}})(3)$$

Since  $e\text{Ch}_t(O_2|A_{\text{BET}}) = e\text{Ch}_t(\langle \text{Heads} \rangle | A_{\text{BET}})$ ,  $e\text{Ch}_t(O_{-3}|A_{\text{BET}}) = e\text{Ch}_t(\sim \langle \text{Heads} \rangle | A_{\text{BET}})$ , and since Ann is rationally certain that taking the bet will not affect the chance of the oddly shaped coin landing heads,  $e\text{Ch}_t(\langle \text{Heads} \rangle | A_{\text{BET}}) = e\text{Ch}_t(\langle \text{Heads} \rangle)$  and  $e\text{Ch}_t(\sim \langle \text{Heads} \rangle | A_{\text{BET}}) = e\text{Ch}_t(\sim \langle \text{Heads} \rangle)$ . Hence,  $U(A_{\text{BET}}) = e\text{Ch}_t(\langle \text{Heads} \rangle)(2) - e\text{Ch}_t(\sim \langle \text{Heads} \rangle)(3)$ . Since  $e\text{Ch}_t(\langle \text{Heads} \rangle)$  and  $e\text{Ch}_t(\sim \langle \text{Heads} \rangle)$  sum to one,  $U(A_{\text{BET}}) \geq U(A_{\text{REFUSE}})$  if and only if  $e\text{Ch}_t(\langle \text{Heads} \rangle) \geq 3/(2+3) = 0.6$ . Since  $e\text{Ch}_t(\langle \text{Heads} \rangle) = 0.5$ , the Rule of U-maximization recommends that Ann refuse the bet. *End of calculation.*

But it seems that Ann clearly should accept the bet. She should be much more than 0.6 confident that the oddly shaped coin will land heads, so she should regard the bet as favorable.

The problem illustrated by *Magical Quartz* generalizes. Suppose that we have some counterexample to the Present Principle, and suppose that for some particular proposition,  $P$ ,  $C_t(P) \neq e\text{Ch}_t(P)$ . Say that an agent faces an *independent propositional bet* on the truth-value of  $P$  just if (1) the agent is deciding whether to accept or refuse a bet that wins \$ $X$  if  $P$  and loses \$ $Y$  if  $\sim P$ , and (2) the agent is rationally certain that her acceptance or refusal of the bet will not affect the chance of  $P$ —that is, for any  $x$ ,  $C_t(\langle \text{Ch}_t(P|A_{\text{BET}}) = x \rangle) = C_t(\langle \text{Ch}_t(P|A_{\text{REFUSE}}) = x \rangle)$ .

I am convinced that there is a straightforward connection between rational credence and rational betting, that the following partial decision rule holds without exception:

**The Credence Rule.** If an agent faces an independent propositional bet that wins \$ $X$  if  $P$  and loses \$ $Y$  if  $\sim P$ , it is rationally permissible for the agent to accept the bet if and only if  $C_t(P) \geq Y/(X+Y)$ .

The Rule of U-maximization entails the following rule:

**The Expected Chance Rule.** If an agent faces an independent propositional bet that wins \$ $X$  if  $P$  and loses \$ $Y$  if  $\sim P$ , it is rationally permissible for the agent to accept the bet if and only if  $e\text{Ch}_t(P) \geq Y/(X+Y)$ .

If crystal balls are *impossible*, the Credence Rule and the Expected Chance Rule necessarily coincide, since then it follows by the Principal Principle that  $C_t(P) = e\text{Ch}_t(P)$ . But if crystal balls are *possible* and, in a given case,  $C_t(P) \neq e\text{Ch}_t(P)$ , then

we can choose X and Y so that the Credence Rule and the Expected Chance Rule diverge. *Every such case is a counterexample to the Rule of U-maximization.* The Rule of U-maximization is defensible, I believe, only if it never conflicts with the Credence Rule.

### 7. The New Principle and the Rule of N-maximization

There is a familiar strategy, already on offer, for handling crystal balls. The strategy has agents defer to conditional chances. If a proposition Q is compatible with  $\langle \text{Ch}_t(P|Q) = x \rangle$ , then Q is admissible with respect to  $\langle \text{Ch}_t(P|Q) = x \rangle$ , so there is a chance norm for credence that requires no admissibility clause.<sup>7</sup>

**The New Principle.**  $C_0(P|Q \& \langle \text{Ch}_t(P|Q) = x \rangle) = x$ .

Crystal balls pose no threat to the New Principle. If E is an agent's evidence at  $t$ , then E is admissible with respect to any  $\langle \text{Ch}_t(P|E) = x \rangle$  with which it is compatible, so  $C_t(P) = C_0(P|E) = e\text{Ch}_t(P|E)$ .

If crystal balls are possible, proponents of causal decision theories can use an analog of the New Principle. If E is an agent's total evidence at  $t$ , let the N-value of an option A be  $\sum_O e\text{Ch}_t(O|A \& E)u(O)$ . We then have

**The Rule of N-maximization.** Rationality requires that agents choose so as to maximize N.

Since  $C_t(P) = C_0(P|E) = e\text{Ch}_t(P|E)$ , the Rule of N-maximization never conflicts with the Credence Rule.

But the Rule of N-maximization, like the New Principle, is user-hostile. It involves chances conditionalized on the agent's evidence, which are strange and hard-to-estimate. (What test could we run to determine whether  $\text{Ch}_t(\langle \text{Heads} \rangle | E) = 0.5$ , for example?) In the face of the problem of crystal balls, it would be better if causal decision theorists could defend the Rule of U-maximization—and I think that they can.

### 8. An Argument for the Present Principle

There are five claims that jointly entail the Present Principle, and I think that all five are true.

The first claim is the Principal Principle.

The second claim concerns admissibility. Let  $H_{tw}$  be the  $t$ -history proposition that fully specifies the history of world  $w$  up to time  $t$ . Let  $H_t$  be the set of  $t$ -history propositions. Any proposition that is (necessarily equivalent to) a disjunction of  $t$ -history propositions is a putative bit of  $t$ -historical information. The second claim states that any putative bit of  $t$ -historical information is admissible with respect to the  $t$ -chances:

**Historical Admissibility.** If Q is a disjunction of  $t$ -history propositions, then Q is admissible with respect to any  $t$ -chance proposition with which it is compatible.

The third claim concerns evidence. If  $P$  is entailed by an agent's evidence at  $t$ , then I will say that the agent *possesses*  $P$  at  $t$ . Let  $\langle \text{POS}_t(P) \rangle$  be a proposition that is true just if the agent possesses  $P$  at  $t$ . The third claim states that only true propositions can be possessed:

**Factivity.** For any  $P$ , if  $\langle \text{POS}_t(P) \rangle$ , then  $P$ .

The fourth claim states that evidence is positively accessible:

**Positive Access.** For any proposition  $P$ , if  $\langle \text{POS}_t(P) \rangle$ , then  $\langle \text{POS}_t(\text{POS}_t(P)) \rangle$ .

The fifth claim derives from Ockham. Say that a proposition is *fixed at  $t$*  just if any  $t$ -history proposition entails either it or its negation. The fifth claim states that propositions about what evidence is possessed at  $t$  are always fixed at  $t$ :

**Possessive Fixity.** For every  $\langle \text{POS}_t(P) \rangle$ ,  $\langle \text{POS}_t(P) \rangle$  is fixed at  $t$ .

The argument then proceeds by *reductio*. Suppose that  $E$  is an agent's evidence at  $t$ , and suppose, for *reductio*, that  $E$  is inadmissible with respect to some  $t$ -chance proposition with which it is compatible—say,  $\langle \text{Ch}_t(P) = x \rangle$ . Then, by the definition of admissibility,

$$x \neq C_0(P|E \& \langle \text{Ch}_t(P) = x \rangle).$$

By Positive Access, since  $\langle \text{POS}_t(E) \rangle$ ,  $\langle \text{POS}_t(\langle \text{POS}_t(E) \rangle) \rangle$ . Hence,

$$C_0(P|E \& \langle \text{Ch}_t(P) = x \rangle) = C_0(P|E \& \langle \text{POS}_t(E) \rangle \& \langle \text{Ch}_t(P) = x \rangle).$$

By Possessive Fixity, each  $t$ -history proposition entails either  $\langle \text{POS}_t(E) \rangle$  or  $\sim \langle \text{POS}_t(E) \rangle$ . So, if  $H$  is the disjunction of the  $t$ -history propositions that entail  $\langle \text{POS}_t(E) \rangle$ ,

$$C_0(P|E \& \langle \text{POS}_t(E) \rangle \& \langle \text{Ch}_t(P) = x \rangle) = C_0(P|E \& H \& \langle \text{Ch}_t(P) = x \rangle).$$

Since evidence is factive, each  $t$ -history proposition that entails  $\langle \text{POS}_t(E) \rangle$  also entails  $E$ . So,

$$C_0(P|E \& H \& \langle \text{Ch}_t(P) = x \rangle) = C_0(P|H \& \langle \text{Ch}_t(P) = x \rangle).$$

Since  $H$  is a disjunction of  $t$ -history propositions, it follows by Historical Admissibility that

$$C_0(P|H \& \langle \text{Ch}_t(P) = x \rangle) = C_0(P|\langle \text{Ch}_t(P) = x \rangle).$$

But it follows from the Principal Principle that

$$C_0(P|\langle \text{Ch}_t(P) = x \rangle) = x.$$

We thus derive a contradiction,

$$x \neq x.$$

Therefore, E is *admissible* with respect to  $\langle \text{Ch}_t(\text{P}) = x \rangle$ . *End of argument.*

In effect, the argument alleges that an agent's evidence at  $t$  is always a bit of  $t$ -historical information. If E is an agent's evidence at  $t$ , then, by Positive Access,  $\text{E} \& \langle \text{POS}_t(\text{E}) \rangle$  is also the agent's evidence at  $t$ . By Possessive Fixity,  $\langle \text{POS}_t(\text{E}) \rangle$  is a disjunction of  $t$ -history propositions, H, so  $\text{E} \& \text{H}$  is also the agent's evidence at  $t$ . Each disjunct in H entails E, so H is also the agent's evidence at  $t$ , and H is a bit of  $t$ -historical information.

Note that the argument generalizes. Let  $t+$  be some time after  $t$ . If E is an agent's evidence at  $t$ , and E is compatible with  $\langle \text{Ch}_{t+}(\text{P}) = x \rangle$ , then we have

$$\text{The Future Principle. } C_0(\text{P} | \text{E} \& \langle \text{Ch}_{t+}(\text{P}) = x \rangle) = x.$$

Any putative bit of  $t$ -historical information is a putative bit of  $t+$ -historical information as well, so if the argument above establishes the Present Principle, it also establishes the Future Principle.

Of course, none of the five claims above is entirely uncontroversial. I will not say anything further about Factivity<sup>8</sup> or the Principal Principle, but let me say a bit more about the other three claims.

## 9. Historical Admissibility

On any orthodox conception of chance, neither the past nor the present can be presently chancy. The future may be chancy, but there can be no present chance that the past or present might be any different than they are.

Every orthodox conception of chance entails Historical Admissibility. If the past and present cannot be presently chancy, then, if H is a disjunction of  $t$ -history propositions, the  $t$ -chance of H at any world is either 0 or 1. Hence, it follows by the New Principle that, if H is compatible with  $\langle \text{Ch}_t(\text{P}) = x \rangle$ ,

$$C_0(\text{P} | \text{H} \& \langle \text{Ch}_t(\text{P}) = x \rangle) = C_0(\text{P} | \text{H} \& \langle \text{Ch}_t(\text{P} | \text{H}) = x \rangle) = x.$$

Any orthodox conception of chance thus entails Historical Admissibility.

Anyone who accepts Historical Admissibility must regard cases such as *Magical Quartz* as incoherently described. According to the description of the case, (1) Ann's evidence at  $t$  entails that the  $t$ -chance of the oddly shaped coin landing heads is 0.5, and (2) Ann has excellent reason to trust the deliverances of the crystal ball. But these two claims cannot be both true. It is reasonable for Ann to trust the crystal ball only if it is reasonable for her to believe that the crystal ball is reliable at  $t$ . But if the crystal ball is reliable at  $t$ , then the appearance of the heads image increases the chance of the oddly shaped coin landing heads. This is not to say that the appearance of the heads image *causally influences* how the oddly shaped coin lands, since it may not. The toss might be on the other side of the universe,

outside the future light cone of the crystal ball, locked in an impervious room. But if the crystal ball is reliable at  $t$ , the appearance of the heads image does *increase the chance* of the oddly shaped coin landing heads. A Bayesian argument I owe to Bernhard Salow may help clarify why this is so.<sup>9</sup> Consider an instant of time,  $t_0$ , before any image has appeared in the crystal ball. Let  $\langle \text{Heads} \rangle$  be the proposition that the oddly shaped coin lands heads, and let  $\langle \text{I:Heads} \rangle$  be the proposition that a heads image appears in the crystal ball. By Bayes's Theorem,

$$\text{Ch}_{t_0}(\langle \text{Heads} \rangle | \langle \text{I:Heads} \rangle) = \text{Ch}_{t_0}(\langle \text{I:Heads} \rangle | \langle \text{Heads} \rangle) \text{Ch}_{t_0}(\langle \text{Heads} \rangle) / \text{Ch}_{t_0}(\langle \text{I:Heads} \rangle).$$

Suppose, to make things simple, that every toss of the oddly shaped coin is preceded by an image appearing in the crystal ball, and suppose that before any image has appeared, the chance of  $\langle \text{Heads} \rangle$  and the chance of  $\langle \text{I:Heads} \rangle$  are both equal to 0.5. If the crystal ball is reliable at  $t_0$ , then  $\text{Ch}_{t_0}(\langle \text{I:Heads} \rangle | \langle \text{Heads} \rangle) = n > 0.5$ : the conditional  $t_0$ -chance of a heads image given that the oddly shaped coin lands heads is greater than 0.5. By Bayes's Theorem, then,  $\text{Ch}_{t_0}(\langle \text{Heads} \rangle | \langle \text{I:Heads} \rangle) = n$ : the conditional  $t_0$ -chance of the future coin landing heads given the appearance of a heads image also equals  $n$ . Now, playing time forward, suppose that a heads image appears in the crystal ball at time  $t$ . The appearance of a heads image presumably increases the chance of  $\langle \text{I:Heads} \rangle$  from 0.5 to 1. (Looking at the heads image in the crystal ball, we cannot very well say, "There is a heads image in this crystal ball, yet there is 0.5 (present) chance that there isn't.") But if the  $t$ -chance of  $\langle \text{I:Heads} \rangle$  equals 1, and if the  $t$ -chances update by conditionalization, then the chance of  $\langle \text{Heads} \rangle$  will have increased from 0.5 to  $n$ . The appearance of the heads image thus will have increased the chance of the oddly shaped coin landing heads in the future from 0.5 to  $n$ .

The intuition that *Magical Quartz* is meant to elicit—namely, that Ann should be much more than 0.5 confident that the oddly shaped coin will land heads—is right. But the claim that Ann might thereby possess information not "known" by the present chances is wrong. Ann should be much more than 0.5 confident that the oddly shaped coin will land heads because, having seen the heads image, she should expect the present chance of the oddly shaped coin landing heads to be much more than 0.5.

One can define unorthodox notions of chance, on which the present and past can be presently chancy. In the framework for thinking about chance that I favor, there is a metaphysically prior chance function,  $\text{Ch}_0(\cdot)$ , defined over the space of all possible worlds, and all other chance functions are derived from it via conditionalization.<sup>10</sup> If  $L_w$  is the *law proposition* that fully specifies the laws at world  $w$ , then  $\text{Ch}_0(\cdot | L_w)$  is the nomologically prior chance function at  $w$ . If  $H_{t,w}$  is the  $t$ -history proposition that holds at  $w$ , then  $\text{Ch}_0(\cdot | L_w \& H_{t,w})$  is the  $t$ -chance function at  $w$ .

In this framework, it is analytic that Historical Admissibility holds of  $t$ -chances. The  $t$ -chance function at world  $w$  is always conditionalized on the  $t$ -history proposition that holds at  $w$ , so the  $t$ -chance of any disjunction of  $t$ -history propositions is either 0 or 1. But, of course, nothing forces us to conditionalize on law and/or  $t$ -history propositions. When *de facto* asymmetries between past and future break down, our reasons for caring about chance may splinter.<sup>11</sup> For example, in ordinary

worlds, historicity and counterfactual dependence go together; the present and past do not depend counterfactually on the future. But, in extraordinary worlds, such as the one in which *Magical Quartz* is set, historicity and counterfactual dependence can and do come apart. What image appears in the crystal ball at  $t$  depends counterfactually on the outcome of a coin tossed after  $t$ . Let  $H^-_{tw}$  fully specify every part of the history of world  $w$  up to time  $t$  that does not counterfactually depend on anything after  $t$ . We could call  $\text{Ch}_0(\cdot | L_w \& H^-_{tw})$  the *unalloyed*  $t$ -chance function at  $w$ .<sup>12</sup> If  $H^-_{tw}$  is weaker than  $H_{tw}$ , then  $\text{Ch}_0(H_{tw} | L_w \& H^-_{tw})$  may be nontrivial. The present and past then would have a nontrivial unalloyed  $t$ -chance.

Note that Historical Admissibility fails for the unalloyed  $t$ -chances. In fact, the evidence that Ann possesses at  $t$  by virtue of looking into the crystal ball is inadmissible with respect to the unalloyed  $t$ -chances. After the heads image appears, there remains a 0.5 unalloyed  $t$ -chance that the heads image has not appeared, since its appearance depends counterfactually on the future.

But the fact that there are unalloyed  $t$ -chances, as well as other notions of chance for which Historical Admissibility fails, casts no doubt on the existence of  $t$ -chances, and Historical Admissibility holds for  $t$ -chances. Perhaps there are purposes for which the unalloyed  $t$ -chances are the right chances to focus upon. But when it comes to deference and decision theory, the right chances to focus upon are the  $t$ -chances. Both the Present Principle and the Rule of U-maximization are, as they should be, formulated in terms of  $t$ -chances.

## 10. Possessive fixity

Possessive Fixity, a supervenience thesis, states that what evidence agents possess at time  $t$  supervenes on the history of the world up to  $t$ .

Some conceptions of evidence immediately entail Possessive Fixity. Take the slice-phenomenal conception of evidence, for example, on which an agent's evidence at  $t$  is the strongest proposition entailed by the agent's phenomenal state at  $t$ .<sup>13</sup> Take any agent and any  $t$ -history proposition. For any proposition  $P$ , the  $t$ -history proposition either entails that  $P$  is entailed by the agent's phenomenal state or entails that  $P$  is *not* entailed by the agent's phenomenal state. On the slice-phenomenal conception of evidence,  $\langle \text{POS}_t(P) \rangle$  is the proposition that  $P$  is entailed by the agent's phenomenal state at  $t$ , so the slice-phenomenal conception entails Possessive Fixity.

Of course, much else besides an agent's phenomenal state at  $t$  supervenes on the history of the world up to  $t$ , so other conceptions of evidence entail Possessive Fixity, too. We can add an agent's beliefs and apparent memories (at least those with narrow, as opposed to broad, contents) and still verify Possessive Fixity. I think that many (probably most) evidential internalists should accept Possessive Fixity.

Many externalist conceptions of evidence are hostile to Possessive Fixity. The hostility comes from the combination of two claims. First, it is alleged that some mental attitude,  $M$ , is *evidence generating*: that an agent at  $t$  possesses every proposition to which she is  $M$ -related at  $t$ . Then, it is alleged that an agent, at time  $t$ , can be  $M$ -related to a proposition that is not entailed by the history of the world up to  $t$ .

A  $t$ -history proposition cannot entail  $\langle \text{POS}_t(\text{P}) \rangle$  if it does not entail  $\text{P}$ , so the two claims, taken together, falsify Possessive Fixity.

I want to look at two mental attitudes that are alleged to be evidence generating, knowing and remembering.

### 10.1 *Knowing*

According to Williamson (2000),  $\text{E} = \text{K}$ : an agent's evidence at  $t$  is the conjunction of the propositions she knows at  $t$ .

In principle, one could reconcile Possessive Fixity and  $\text{E} = \text{K}$  by insisting that agents know virtually nothing about the future, but let us assume, anti-skeptically, that agents know quite a lot about the future. At indeterministic worlds, very little about the future is settled by the past, so if an agent thinks that the actual world might be indeterministic, almost everything the agent knows about the future will be presently inadmissible. For example, take an agent who knows that lightning is chancy:<sup>14</sup>

*Lightning.* At time  $t$ , Ben knows that in two months' time, his parents will be staying in his house. It is the stormy season, and Ben knows that lightning is chancy. Ben thus gives nonzero credence to there being a nonzero chance of his house being destroyed (permanently) by a bolt of lightning in the next month.

Let  $\langle \text{Bolt} \rangle$  be the proposition that Ben's house is destroyed by a bolt of lightning in the next month. According to *Lightning*, Ben, at time  $t$ , knows  $\sim \langle \text{Bolt} \rangle$ ,<sup>15</sup> so, if  $\text{E} = \text{K}$ , Ben's evidence at  $t$  entails  $\sim \langle \text{Bolt} \rangle$ . The history of the world up to time  $t$  does not entail either  $\langle \text{Bolt} \rangle$  or  $\sim \langle \text{Bolt} \rangle$ , however. It is a chancy matter, at time  $t$ , whether Ben's house will be destroyed by a bolt of lightning. Thus, if  $\text{E} = \text{K}$ , *Lightning* is a counterexample to Possessive Fixity. Although  $\sim \langle \text{Bolt} \rangle$  is possessed by Ben at  $t$ , the history of the world up to time  $t$  does not entail that  $\sim \langle \text{Bolt} \rangle$  is possessed by Ben at  $t$ .

If *Lightning* is a counterexample to Possessive Fixity, it is also a counterexample to the Present Principle. Let  $\text{E}$  be Ben's evidence at  $t$ . Ben gives nonzero credence to there being a nonzero  $t$ -chance that his house is destroyed by a bolt of lightning in the next month, so there is some  $x$ ,  $0 < x < 1$ , such that  $\langle \text{Ch}_t(\langle \text{Bolt} \rangle) = x \rangle$  is compatible with  $\text{E}$ . But, *contra* the Present Principle,  $\text{C}_0(\langle \text{Bolt} \rangle | \text{E} \& \langle \text{Ch}_t(\langle \text{Bolt} \rangle) = x \rangle) = 0$ , not  $x$ , since  $\text{E}$  entails  $\sim \langle \text{Bolt} \rangle$ .

Another example is even simpler:<sup>16</sup>

*10 Coins.* Cate knows that ten fair and chancy coins will be tossed tomorrow. She also knows that at least one of the coins will land tails—not because she knows something special about any of the coins but because her belief is safe and true.<sup>17</sup>

If  $\text{E} = \text{K}$ , then *10 Coins* is a counterexample to Possessive Fixity. At time  $t$ , Cate knows  $\sim \langle 10\text{Heads} \rangle$ , so, if  $\text{E} = \text{K}$ ,  $\sim \langle 10\text{Heads} \rangle$  is entailed by her evidence at  $t$ . The history of the world up to time  $t$  does not entail  $\sim \langle 10\text{Heads} \rangle$ , however. It is a chancy matter, at time  $t$ , whether all ten coins will land heads. So, although  $\sim \langle 10\text{Heads} \rangle$

is possessed by Cate at  $t$ , the history of the world up to time  $t$  does not entail that  $\sim\langle 10\text{Heads}\rangle$  is possessed by Cate at  $t$ .

If *10 Coins* is a counterexample to Possessive Fixity, it is also a counterexample to the Present Principle. Although E entails  $\langle \text{Ch}_t(\langle 10\text{Heads}\rangle) = 1/2^{10} \rangle$ ,  $C_0(\langle 10\text{Heads}\rangle | E \& \langle \text{Ch}_t(\langle 10\text{Heads}\rangle) = 1/2^{10} \rangle) = 0$ , not  $1/2^{10}$ , since E also entails  $\sim\langle 10\text{Heads}\rangle$ .

If *Lightning* and *10 Coins* are counterexamples to Possessive Fixity, then they can be transformed into counterexamples to the Rule of U-maximization. Take *10 Coins*. Suppose that a bet wins \$5 if  $\sim\langle 10\text{Heads}\rangle$  and loses \$9,995 if  $\langle 10\text{Heads}\rangle$ . If E is Cate's evidence at  $t$  and E entails  $\sim\langle 10\text{Heads}\rangle$ , then  $C_t(\sim\langle 10\text{Heads}\rangle | E) = 1$ . The Credence Rule thus recommends that Cate accept the bet—indeed, the Credence Rule recommends that Cate accept a bet that gains \$5 if  $\sim\langle 10\text{Heads}\rangle$  and loses \$9,995,000 if  $\langle 10\text{Heads}\rangle$ ! However, the Rule of U-maximization recommends that Cate *refuse* the bet. The relevant calculations are as follows:

$$\begin{aligned} U(A_{\text{REFUSE}}) &= \sum_{\text{O}} e\text{Ch}_t(\text{O} | A_{\text{REFUSE}}) u(\text{O}) \\ &= e\text{Ch}_t(\sim\langle 10\text{Heads}\rangle | A_{\text{REFUSE}})(0) - e\text{Ch}_t(\langle 10\text{Heads}\rangle | A_{\text{REFUSE}})(0) = 0 \end{aligned}$$

$$\begin{aligned} U(A_{\text{BET}}) &= \sum_{\text{O}} e\text{Ch}_t(\text{O} | A_{\text{BET}}) u(\text{O}) \\ &= e\text{Ch}_t(\sim\langle 10\text{Heads}\rangle | A_{\text{BET}})(5) - e\text{Ch}_t(\langle 10\text{Heads}\rangle | A_{\text{BET}})(9,995) \\ &= e\text{Ch}_t(\sim\langle 10\text{Heads}\rangle)(5) - e\text{Ch}_t(\langle 10\text{Heads}\rangle)(9,995) \\ &= (1 - (1/2^{10}))(5) - (1/2^{10})(9,995) \approx -5. \end{aligned}$$

The Credence Rule holds without exception, so, if  $E = K$ , *10 Coins* is a counterexample to the Rule of U-maximization.

The same applies to *Lightning*. Suppose that for a small fee, Ben can purchase a favorable insurance policy that pays out just if his house is destroyed by a bolt of lightning in the next month. If the policy is favorable enough, the Rule of U-maximization recommends that Ben purchase the policy. But if  $\sim\langle \text{Bolt}\rangle$  is entailed by Ben's evidence at  $t$ , then Ben, if he is rational, is certain that his house will not be destroyed by a bolt of lightning in the next month, even though he regards lightning as a chancy matter. The Credence Rule thus recommends that Ben not purchase the policy.

It is worth pausing at this point to note something important. Disputes about causal decision theory often involve *recherché* cases in which there are predictors, time travelers, spheres made of magical quartz, or other philosophical fantasies. But if knowledge is evidence generating and evidence plays the certainty-licensing role, then ordinary, everyday agents possess lots of presently inadmissible evidence. Ordinary, everyday decisions, therefore, *often* will be counterexamples to the Rule of U-maximization. Strictly speaking, these are crystal ball cases, since they involve agents with information about the as-yet-undetermined parts of the future. But the label is misleading. The cases are not fantastic. There is nothing arcane about

buying an insurance policy. A decision theory that is incapable of handling ordinary, everyday decisions is in deep, *deep* trouble.

If knowledge is evidence generating, causal decision theorists will have to reject the Rule of U-maximization and turn to the Rule of N-maximization. This would be a cost, as I see it, but not a decisive failing.

However, I am doubtful that *Lightning* and *10 Coins* really are counterexamples to the Rule of U-maximization because it seems to me that the Rule of U-maximization gives the correct recommendation in both cases: that Cate should refuse the bet, and that Ben should purchase the policy. In my view, the wrong recommendations come from the Credence Rule combined with the claim that knowledge is evidence generating. Since I am independently convinced that the Credence Rule cannot admit of counterexamples—I think that rational credence and rational betting are too intimately related to ever come apart—I blame the externalism. In the context of decision theory, evidence should play the certainty-licensing role,<sup>18</sup> and if evidence plays the certainty-licensing role, I say, knowledge is not evidence generating.<sup>19</sup>

If knowing is *not* evidence generating, then neither *Lightning* nor *10 Coins* pose any threat to Possessive Fixity, the Present Principle, or the Rule of U-maximization.

## 10.2 Remembering

Our original example, *Magical Quartz*, fails as a counterexample to the Present Principle because the bit of history that is thought to potentially give Ann information about the as-yet-undetermined parts of the future lies in the past. If Historical Admissibility holds, the present chances “know” everything about the past, so no evidence gained from the past can be news to the present chances.

A more serious threat is posed by cases in which the relevant bit of history lies in the *future*. These cases, which we can construct by combining memory and backward time travel, threaten the Present Principle by threatening Possessive Fixity. Let  $t$  be some time in the year 2020, and consider the following two possible worlds, which are exactly alike up to  $t$ :

*Travel.* Tele was born in the year 3000. In 3030, a lottery is held. Tele learns, and forms a memory, that ticket #852,025,860 wins. Tele then gets into a time machine and travels backward to the year 2020, arriving at time  $t$ .

*Creation.* An intrinsic duplicate of Tele and an intrinsic duplicate of Tele’s time machine are created *ex nihilo* at time  $t$ . The duplicate—Twele—has a vivid apparent memory of having seen a lottery drawn in the year 3030, having found out that ticket #852,025,860 wins, having stepped into a time machine, and having traveled back to the year 2020, arriving at time  $t$ .

Let  $E_{\text{TELE}}$  be Tele’s evidence at  $t$ , and let  $E_{\text{TWELE}}$  be Twele’s. If Possessive Fixity is true, then  $E_{\text{TELE}} = E_{\text{TWELE}}$ .

One might try to argue that  $E_{\text{TELE}} \neq E_{\text{TWELE}}$  by appealing to broad content. Take  $\langle 852,025,860 \rangle$ , the proposition that ticket #852,025,860 wins the lottery in 3030. Tele assigns some credence to  $\langle 852,025,860 \rangle$ , but, arguably, Twele does not.

Twele is not acquainted with years or with lotteries or with tickets, so it seems that Twele cannot have beliefs—either full or partial beliefs—in propositions about particular tickets winning lotteries.

But broad content is a red herring. We should interpret  $C_0(\cdot|E)$ , not as the credence function had by a perfectly rational agent, but as a function that maps each proposition to the credence that the agent is rational in having in that proposition, irrespective of whether the agent has a credence in that proposition. A mere difference in which propositions Tele and Twele have credences in will not refute Possessive Fixity.<sup>20</sup>

The important question is epistemological. Do  $E_{TELE}$  and  $E_{TWELE}$  support  $\langle 852,025,860 \rangle$  to exactly the same degree? Clearly not, if remembering is evidence generating. Tele remembers that ticket #852,025,860 wins the lottery in 3030, and Twele does not. Indeed,  $\langle 852,025,860 \rangle$  could be false at the world that Twele inhabits.

That being said, however, I am convinced, on the basis of rational betting behavior, that  $E_{TELE}$  and  $E_{TWELE}$  *do* support  $\langle 852,025,860 \rangle$  equally. Consider every independent propositional bet on the truth-value of  $\langle 852,025,860 \rangle$ . If  $E_{TELE}$  supports  $\langle 852,025,860 \rangle$  more than  $E_{TWELE}$  does, then there should be a bet that wins \$X if  $\langle 852,025,860 \rangle$  and loses \$Y if  $\sim\langle 852,025,860 \rangle$  that Tele should accept and Twele should reject. But try to find one! There does not seem to be one. Rather, it seems that Tele and Twele should accept and reject exactly the same independent propositional bets on the truth-value of  $\langle 852,025,860 \rangle$ , which strongly supports the claim that  $E_{TELE} = E_{TWELE}$ .

Pairs of cases such as *Travel* and *Creation* pose the greatest threat to Possessive Fixity. If  $E_{TELE} = E_{TWELE}$ , then Possessive Fixity appears defensible.

## 11. Positive Access

Together, Historical Admissibility, Factivity, and Possessive Fixity entail that none of the information possessed by any agent at  $t$  is news to the  $t$ -chances. But, interestingly, this fact does not entail that it is impossible for an agent's evidence at  $t$  to be presently inadmissible. The following example illustrates why:

*Flicker.* Dawn knows that a die will be rolled tomorrow. Dawn, who knows herself to be a perfectly reliable clairvoyant, has a brief mental episode, a mental flicker. Dawn cannot tell whether the mental flicker is a clairvoyant image of the future die landing six or a mild seizure. (In fact, the mental flicker is a clairvoyant image and, as a result, Dawn's evidence entails that the future die will land six.)

Let  $\langle 6 \rangle$  be the proposition that the future die lands six. If Dawn is rational, then, since her evidence at  $t$  entails  $\langle 6 \rangle$ ,  $C_t(\langle 6 \rangle) = 1$ . But Dawn is uncertain about whether  $\langle 6 \rangle$  is entailed by her evidence. Some of her credence is assigned to  $\langle \text{POS}_t(\langle 6 \rangle) \rangle$ -worlds, at which her mental flicker is a clairvoyant image, and some is assigned to  $\sim\langle \text{POS}_t(\langle 6 \rangle) \rangle$ -worlds, at which the mental flicker is a mild seizure. If Historical Admissibility, Factivity, and Possessive Fixity hold, then at every  $\langle \text{POS}_t(\langle 6 \rangle) \rangle$ -world—that is, at every world where the mental flicker is a clairvoyant image—the  $t$ -chance

of (6) is 1. The clairvoyant image raises the chance of the die landing six, just as the heads image raises the chance of the oddly shaped coin landing heads. But the  $t$ -chance at the  $\sim\langle\text{POS}_t((6))\rangle$ -worlds to which Dawn assigns nonzero credence is  $1/6$ . At those worlds, the die is chancy and fair. Dawn's evidence at  $t$  is therefore presently inadmissible. If  $E$  is her evidence at  $t$ , then although  $E$  is compatible with  $\langle\text{Ch}_t((6)) = 1/6\rangle$ ,  $C_0(\langle 6 \rangle | E \& \langle\text{Ch}_t((6)) = 1/6\rangle) = 1$ , not  $1/6$ . If *Flicker* is coherently described, Positive Access fails.

I think that *Flicker* is incoherently described and that the case for Positive Access is very strong indeed. It has been proven, as a sort of generalization of Good's Theorem (1966),<sup>21</sup> that if agents should always regard the prospect of gaining evidence as having non-negative epistemic value, Positive Access must hold. I think that the hallmark of something being evidence is its having non-negative epistemic value, so I am therefore very much inclined to accept Positive Access.

Moreover, as Dorst (MSa) has pointed out, it seems that we can reduce putative failures of Positive Access to absurdity. If  $E$  is an agent's evidence at  $t$  and  $E$  does not entail  $\langle\text{POS}_t(E)\rangle$ , then  $C_0(E | E \& \sim\langle\text{POS}_t(E)\rangle) = 1$ . But it seems irrational for an agent to be certain that  $E$ , on the condition that the agent's evidence does not entail  $E$ . We can make the irrationality vivid if we think about betting. Suppose that we offer the agent a conditional bet. If  $\langle\text{POS}_t(E)\rangle$ , the bet is off; if  $\sim\langle\text{POS}_t(E)\rangle$ , the agent wins \$1 if  $E$  and loses \$10 (or \$1,000 or \$1,000,000,000) if  $\sim E$ . A rational agent will not accept this bet. The agent is certain that the bet is on only if she is *irrationally* certain that  $E$ , and it is not rational for an agent to bet at unfavorable odds that  $E$  is true on the condition that she is irrationally certain that  $E$ . But to ensure that rational agents will refuse conditional bet of this sort, Positive Access must hold and  $C_0(E | E \& \sim\langle\text{POS}_t(E)\rangle)$  always must be undefined.

## 12. Conclusion

Lewis (1981) was unsure whether causal decision theory could handle crystal ball cases, so he set such cases aside. In a 1982 letter written to Wlodek Rabinowicz, Lewis revisits and reiterates his concern:

[...] I believe in the logical possibility of time travel, precognition, etc., and I see no reason why suitable evidence might not convince a perfectly rational agent that these possibilities are realized, and in such a way as to bring him news from the future. My worry is a different one. It seems to me completely unclear what conduct would be rational for an agent in such a case. [...] You know that spending all you have on armour would greatly increase your chances of surviving the coming battle, but leave you a pauper if you do survive; but you also know, by news of the future that you have excellent reasons to trust, that you will survive. (The news doesn't say whether you will have bought the armour.) Now: is it rational to buy the armour? I have no idea—there are excellent reasons both ways. And I think that even those who have the correct two-box'ist intuitions about Newcomb's problem may still find this new

problem puzzling. That is, I *don't* think that the appeal of not buying the armour is just a misguided revival of [one-box'ist] intuitions that we've elsewhere overcome.

Recent critics of causal decision theory maintain that it is illegitimate to set crystal ball cases aside.<sup>22</sup> They seem mainly concerned with dialectical parity. Newcomb problems are strange cases that motivate accepting causal decision theory. Crystal ball cases, they think, are strange cases that motivate rejecting causal decision theory. Using Newcomb problems but ignoring crystal ball cases seems to them a tendentious way of stacking the deck.

I also think that it is illegitimate to set crystal ball cases aside, but for a different reason. I worry mainly about prevalence. Lewis thinks that he is “only setting aside some very special cases” (1981, 18). But it is unclear how special crystal ball cases are. If failures of Possessive Fixity are widespread, then so too are crystal ball cases; the “extraordinary case of an agent who thinks he may somehow have foreknowledge of the outcomes of chance processes” (1981, 18) is then the *ordinary* case of an agent making future plans, buying an insurance policy, deciding how much to save for the future, and so on. Fantastic decisions perhaps can be set aside legitimately, but ordinary decisions cannot be. In this way, I think that previous causal decision theorists have *underestimated* the problem of crystal balls.

Of course, there is also a way in which they have *overestimated* it. They think that crystal ball cases are possible. Lewis, for example, thinks that the decision whether to buy the armor is a crystal ball case. If my argument above is sound, Lewis is wrong. Crystal ball cases are not possible. The decision whether to buy the armor is incoherently described.

At time  $t$ -, before the agent receives the testimony from the time traveler, it may be true that buying the armor would greatly increase her chance of survival. Let  $\langle \text{Survive} \rangle$  be the proposition that the agent survives the battle, and let  $A_{\text{BUY}}$  and  $A_{\text{REFUSE}}$  be the options of buying and refusing to buy the armor, respectively. It may be true, then, that  $\text{Ch}_t(\langle \text{Survive} \rangle | A_{\text{BUY}})$  is high and  $\text{Ch}_t(\langle \text{Survive} \rangle | A_{\text{REFUSE}})$  is low. But the agent receiving reliable testimony that she will survive the coming battle also greatly increases her chance of survival. Receiving the testimony may not cause the agent to survive, but, like many effects that predate their causes, it does increase the chance. Let  $\langle \text{T:Survive} \rangle$  be the proposition that the reliable time traveler tells the agent that she will survive the coming battle. Then  $\text{Ch}_t(\langle \text{Survive} \rangle | A_{\text{REFUSE}} \& \langle \text{T:Survive} \rangle)$  is high, not low, and  $\text{Ch}_t(\langle \text{Survive} \rangle | A_{\text{BUY}} \& \langle \text{T:Survive} \rangle)$  is at most only slightly higher. In other words, at time  $t$ , when the agent is deciding whether to purchase the armor, having already received the news from the future, it is not true that buying the armor greatly increases her chance of survival. Therefore, it is not rational to buy the armor. The costs are exorbitant and the added safety is negligible.<sup>23</sup>

As Lewis rightly surmised, “the appeal of not buying the armour is [not] just a misguided revival of [one-box'ist] intuitions that we've elsewhere overcome.” Two-boxing in Newcomb's problem uniquely maximizes U, but so too does marching into battle well heeled and unarmored.<sup>24</sup>

## Notes

<sup>1</sup> See, among many others, Eagle (2014), Elliott (2016), Hall (1994), Hoefer (2007; 2014), Lewis (1980; 1994), Roberts (2001), Strevens (1999), Vranas (2004), and Williamson (2009). Ismael (2008) and Joyce (2007) doubt that crystal balls are physically possible. Like me, Meacham (2010) doubts that crystal balls are metaphysically possible.

<sup>2</sup> See, for example, Bales (2016), Lewis (1981), Rabinowicz (1982; 2009), and Sobel (1978 [1980]). Egan uses crystal ball cases primarily to highlight decision instability. Even if crystal ball cases are impossible, decision instability remains problematic. In Spencer (MS), I propose a form of causal decision theory that I believe can handle decision instability. In what follows, I set decision instability aside.

<sup>3</sup> Lewis characterizes admissible evidence as “information whose impact on credence about outcomes comes entirely by way of credences about the chances of those outcomes” (1980, 272). My characterization is slightly broader. It includes any evidence that, when added to what Lewis counts as admissible, has no effect on the agent’s credence in the chance proposition. The broader characterization simplifies some of the calculations, but nothing of substance turns on which characterization we use.

<sup>4</sup> Part of the disagreement stems from the disagreement between Humeans and anti-Humeans. Anti-Humeans can agree about how to define admissibility (cf. Meacham 2010). Let  $\mathbf{LH}_t$  contain every possible combination of law and  $t$ -history propositions. Then the following definition of admissibility is one that anti-Humeans can accept:  $Q$  is admissible with respect to  $\langle \text{Ch}_t(P)=x \rangle$  if and only if  $Q \& \langle \text{Ch}_t(P)=x \rangle$  is a disjunction of members of  $\mathbf{LH}_t$ .

<sup>5</sup> A causal decision theory formulated in terms of counterfactuals is committed to counterfactual excluded middle. The apparent counterexamples to counterfactual excluded middle include crystal ball cases, such as *Magical Quartz*, which is one reason to favor formulations in terms of chance; cf. Lewis (1981) and Joyce (1999). The quantum mechanical cases in Ahmed (2014, ch. 6) provide another reason to favor chance-based formulations.

<sup>6</sup> If agents can know that an option has no chance of being chosen, then we should follow Lewis (1981) and formulate the Rule of U-maximization using counterfactual chances.

<sup>7</sup> Cf. Hall (1994; 2004) and Lewis (1994).

<sup>8</sup> For arguments against Factivity, see Comensaña and McGrath (2014) and Fitelson (2017).

<sup>9</sup> Cf. Salow (MS).

<sup>10</sup> Moreover, I am inclined to think that  $\text{Ch}_0(\cdot) = C_0(\cdot)$ .

<sup>11</sup> Cf. Lewis (1980). Lewis has a second concern about Historical Admissibility, that odd patterns of entropy or atypical spacetime geometries might make it impossible to define a globally applicable arrow of time. This concern can be assuaged, so long as there is a well-defined arrow of time at each spacetime point. Let  $p$  be a spacetime point, and let  $H_{pw}$  be a full specification of how world  $w$  is outside of the future light cone of  $p$ . The  $p$ -chance function at  $w$  is  $\text{Ch}_0(\cdot | L_w H_{pw})$ . A  $p$ -chance proposition  $\langle \text{Ch}_p(P)=x \rangle$  specifies the chance, at  $p$ , of  $P$ . The Principal Principle can be recast in terms of  $p$ -chance; causal decision theory can be recast to recommend that an agent, at spacetime point  $p$ , choose so as to maximize the expected  $p$ -chance of good outcomes; and the argument in section 8 can be recast as an argument that it is impossible for an agent, at  $p$ , to possess evidence that is inadmissible with respect to a  $p$ -chance proposition that is compatible with her evidence.

<sup>12</sup> Cusbert (2018) can be read as an argument that *the* chances are something akin to the unalloyed chances. He notes that the unalloyed chances are not particularly useful in epistemology or decision theory. I doubt that there is any such thing as *the* chances.

<sup>13</sup> See, for example, Lewis (1996). Also see Hedden (2015) and Moss (2015) and their defense of time-slice rationality.

<sup>14</sup> Cf. Hawthorne (2005).

<sup>15</sup> Here I assume single-premise closure. The following example does not require closure.

<sup>16</sup> Relevant discussions of knowledge and chance include Bacon (2014), Dorr, Goodman, and Hawthorne (2014), Goodman and Salow (2018), Hawthorne and Lasonen-Aarnio (2009), and Williamson (2009).

<sup>17</sup> We could increase the number of coins from 10 to 10 million.

<sup>18</sup> Shulz (2017) argues that when stakes are high, evidence does not play the certainty-licensing role even within the context of decision theory. Shulz's conception of evidence violates Positive Access, however, and is for that reason unsatisfactory.

<sup>19</sup> Williamson (2000, ch. 10) agrees but thinks that evidence should not play the certainty-licensing role. For more on  $E=K$  and certainty, see, for example, Clarke (2013), Greco (2013; 2015), Kaplan (2009), and Williamson (2005; 2009).

<sup>20</sup> There is an issue about broad content that is interesting, although tangential. The credence that we assign to broad contents are guise-specific. Perhaps  $\langle \text{Hesperus}=\text{Hesperus} \rangle = \langle \text{Hesperus}=\text{Phosphorus} \rangle$ , but the Babylonians assigned a higher credence to the proposition under the 'H=H' guise than they did to under the 'H=P' guise. Let  $P_G$  be the credence that an agent assigns to P under the G-guise. Let  $\langle \text{Of}(G,P) \rangle$  be the proposition that the G is a guise of P. Then, drawing on Chalmers (2006), we might have the following guise-specific version of the Principal Principle: if  $E \& \langle \text{Ch}_r(P)=x \rangle \& \langle \text{Of}(G,P) \rangle$  is possibly true, and  $E \& \langle \text{Of}(G,P) \rangle$  is admissible with respect to  $\langle \text{Ch}_r(P)=x \rangle$ , then rationality requires that  $C_0(P_G | E \& \langle \text{Ch}_r(P)=x \rangle \& \langle \text{Of}(G,P) \rangle) = x$ . This guise-specific version avoids the counterexamples to the Principal Principle based on clever ways of naming, discussed in Hawthorne and Lasonen-Aarnio (2009). In the relevant examples, even if E is admissible,  $E \& \langle \text{Of}(G,P) \rangle$  is not admissible, since  $E \& \langle \text{Ch}_r(P)=x \rangle \& \langle \text{Of}(G,P) \rangle$  is not a disjunction of the members of  $\mathbf{LH}_r$ . (See fn. 3.) Thanks to David Builes for several helpful discussions on this topic.

<sup>21</sup> Cf. Dorst (MSa) and Geanakoplos (1989). Also see Ahmed and Salow (forthcoming) and Das (MS).

<sup>22</sup> See Price (2012, 504, n. 15). Also see Egan (2007) and Rabinowicz (1982; 2009).

<sup>23</sup> If the time traveler is not reliable, the testimony will increase the chance of survival little or none; it then would be rational to *buy* the armor.

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